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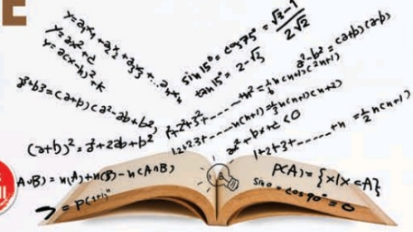

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SOLVED PAPER

Held on
5th Sep, Evening Shift

JEE MAIN 2020

1. If the system of linear equations
$x + y + 3z = 0$; $x + 3y + k^2z = 0$; $3x + y + 3z = 0$
has a non-zero solution (x, y, z) for some $k \in R$, then $x + \left(\frac{y}{z}\right)$ is equal to

(a) -3 (b) 9 (c) 3 (d) -9

2. If α and β are the roots of the equation, $7x^2 - 3x - 2 = 0$, then the value of $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2}$ is equal to

(a) $\frac{27}{32}$ (b) $\frac{1}{24}$ (c) $\frac{3}{8}$ (d) $\frac{27}{16}$

3. If $x = 1$ is a critical point of the function $f(x) = (3x^2 + ax - 2 - a)\,e^x$, then

(a) $x = 1$ and $x = -\frac{2}{3}$ are local minima of f.

(b) $x = 1$ and $x = -\frac{2}{3}$ are local maxima of f.

(c) $x = 1$ is a local maxima and $x = -\frac{2}{3}$ is a local minima of f.

(d) $x = 1$ is a local minima and $x = -\frac{2}{3}$ is a local maxima of f.

4. The area (in sq. units) of the region
$A = \{(x, y) : (x-1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2\}$,
where $[t]$ denotes the greatest integer function, is

(a) $\frac{8}{3}\sqrt{2} - \frac{1}{2}$ (b) $\frac{4}{3}\sqrt{2} + 1$

(c) $\frac{8}{3}\sqrt{2} - 1$ (d) $\frac{4}{3}\sqrt{2} - \frac{1}{2}$

5. If the sum of the second, third and fourth terms of a positive term G.P. is 3 and the sum of its sixth, seventh and eighth terms is 243, then the sum of the first 50 terms of this G.P. is

(a) $\frac{1}{26}(3^{49} - 1)$ (b) $\frac{1}{26}(3^{50} - 1)$

(c) $\frac{2}{13}(3^{50} - 1)$ (d) $\frac{1}{13}(3^{50} - 1)$

6. The value of $\left(\frac{-1 + i\sqrt{3}}{1 - i}\right)^{30}$ is

(a) -2^{15} (b) $2^{15}i$ (c) $-2^{15}i$ (d) 6^5

7. If $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$ and
$M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$, then

(a) $L = -\frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$ (b) $L = \frac{1}{4\sqrt{2}} - \frac{1}{4}\cos\frac{\pi}{8}$

(c) $M = \frac{1}{4\sqrt{2}} + \frac{1}{4}\cos\frac{\pi}{8}$ (d) $M = \frac{1}{2\sqrt{2}} + \frac{1}{2}\cos\frac{\pi}{8}$

8. If $a + x = b + y = c + z + 1$, where a, b, c, x, y, z are non-zero distinct real numbers, then $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$ is equal to

(a) $y(b - a)$ (b) $y(a - b)$
(c) 0 (d) $y(a - c)$

9. If the line $y = mx + c$ is a common tangent to the hyperbola $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and the circle $x^2 + y^2 = 36$, then which one of the following is true?

(a) $c^2 = 369$ (b) $5m = 4$
(c) $4c^2 = 369$ (d) $8m + 5 = 0$

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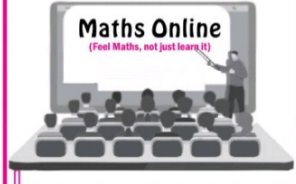

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10. Which of the following points lies on the tangent to the curve $x^4e^y + 2\sqrt{y+1} = 3$ at the point (1, 0)?

(a) (2, 2) (b) (2, 6)
(c) (−2, 6) (d) (−2, 4)

11. The statement $(p \to (q \to p)) \to (p \to (p \vee q))$ is

(a) equivalent to $(p \wedge q) \vee (\sim q)$
(b) a contradiction
(c) equivalent to $(p \vee q) \wedge (\sim p)$
(d) a tautology

12. $\lim\limits_{x\to 0} \dfrac{x\left(e^{\left(\sqrt{1+x^2+x^4}-1\right)/x}-1\right)}{\sqrt{1+x^2+x^4}-1}$,

(a) is equal to $\sqrt{e}$ (b) is equal to 1
(c) is equal to 0 (d) does not exist

13. If the sum of the first 20 terms of the series $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x + \ldots$ is 460, then x is equal to

(a) 7^2 (b) $7^{1/2}$ (c) e^2 (d) $7^{46/21}$

14. The derivative of $\tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\dfrac{2x\sqrt{1-x^2}}{1-2x^2}\right)$ at $x = \dfrac{1}{2}$ is

(a) $\dfrac{2\sqrt{3}}{5}$ (b) $\dfrac{\sqrt{3}}{12}$ (c) $\dfrac{2\sqrt{3}}{3}$ (d) $\dfrac{\sqrt{3}}{10}$

15. If $\int \dfrac{\cos\theta}{5+7\sin\theta-2\cos^2\theta}\,d\theta = A\log_e|B(\theta)| + C$, where C is a constant of integration, then $\dfrac{B(\theta)}{A}$ can be

(a) $\dfrac{2\sin\theta+1}{\sin\theta+3}$ (b) $\dfrac{2\sin\theta+1}{5(\sin\theta+3)}$
(c) $\dfrac{5(\sin\theta+3)}{2\sin\theta+1}$ (d) $\dfrac{5(2\sin\theta+1)}{\sin\theta+3}$

16. Let $y = y(x)$ be the solution of the differential equation $\cos x\dfrac{dy}{dx} + 2y\sin x = \sin 2x,\ x \in \left(0, \dfrac{\pi}{2}\right)$.
If $y(\pi/3) = 0$, then $y(\pi/4)$ is equal to

(a) $2-\sqrt{2}$ (b) $2+\sqrt{2}$ (c) $\sqrt{2}-2$ (d) $\dfrac{1}{\sqrt{2}}-1$

17. If the length of the chord of the circle, $x^2 + y^2 = r^2$ $(r > 0)$ along the line, $y - 2x = 3$ is r, then r^2 is equal to

(a) $\dfrac{9}{5}$ (b) 12 (c) $\dfrac{24}{5}$ (d) $\dfrac{12}{5}$

18. If the mean and the standard deviation of the data 3, 5, 7, a, b are 5 and 2 respectively, then a and b are the roots of the equation

(a) $x^2 - 10x + 18 = 0$ (b) $2x^2 - 20x + 19 = 0$
(c) $x^2 - 10x + 19 = 0$ (d) $x^2 - 20x + 18 = 0$

19. If for some $\alpha \in R$, the lines $L_1 : \dfrac{x+1}{2} = \dfrac{y-2}{-1} = \dfrac{z-1}{1}$ and $L_2 : \dfrac{x+2}{\alpha} = \dfrac{y+1}{5-\alpha} = \dfrac{z+1}{1}$ are coplanar, then the line L_2 passes through the point

(a) (10, 2, 2) (b) (2, −10, −2)
(c) (10, −2, −2) (d) (−2, 10, 2)

20. There are 3 sections in a question paper and each section contains 5 questions. A candidate has to answer a total of 5 questions, choosing at least one question from each section. Then the number of ways, in which the candidate can choose the questions, is

(a) 3000 (b) 1500 (c) 2255 (d) 2250

21. The coefficient of x^4 in the expansion of $(1 + x + x^2 + x^3)^6$ in powers of x, is _____ .

22. In a bombing attack, there is 50% chance that a bomb will hit the target. At least two independent hits are required to destroy the target completely. Then the minimum number of bombs, that must be dropped to ensure that there is at least 99% chance of completely destroying the target, is _____ .

23. If the lines $x + y = a$ and $x - y = b$ touch the curve $y = x^2 - 3x + 2$ at the points where the curve intersects the x-axis, then $\dfrac{a}{b}$ is equal to _____ .

24. Let the vectors $\vec{a}, \vec{b}, \vec{c}$ be such that $|\vec{a}| = 2, |\vec{b}| = 4$ and $|\vec{c}| = 4$. If the projection of $\vec{b}$ on $\vec{a}$ is equal to the projection of $\vec{c}$ on $\vec{a}$ and $\vec{b}$ is perpendicular to $\vec{c}$, then the value of $|\vec{a} + \vec{b} - \vec{c}|$ is _____ .

25. Let $A = \{a, b, c\}$ and $B = \{1, 2, 3, 4\}$. Then the number of elements in the set $C = \{f : A \to B \mid 2 \in f(A)$ and f is not one-one$\}$ is _____ .

SOLUTIONS

1. (a) : We have, system of equations has non-zero solution.

$$\therefore \quad \Delta = \begin{vmatrix} 1 & 1 & 3 \\ 1 & 3 & k^2 \\ 3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 1(9 - k^2) - 1(3 - 3k^2) + 3(1 - 9) = 0$
$\Rightarrow -18 + 2k^2 = 0 \Rightarrow k^2 = 9$

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So, given linear equations become

$x + y + 3z = 0$ ...(i)

$x + 3y + 9z = 0$ ...(ii)

$3x + y + 3z = 0$ ...(iii)

Subtracting (i) from (iii), we get

$2x = 0 \Rightarrow x = 0$

From (i), $y + 3z = 0 \Rightarrow \frac{y}{z} = -3$

$\therefore \quad x + \frac{y}{z} = 0 - 3 = -3$

2. (d) : α, β are roots of given equation $7x^2 - 3x - 2 = 0$.

$\therefore \quad \alpha + \beta = \frac{3}{7}$ and $\alpha\beta = \frac{-2}{7}$

Now, $\frac{\alpha}{1-\alpha^2} + \frac{\beta}{1-\beta^2} = \frac{\alpha(1-\beta^2)+\beta(1-\alpha^2)}{(1-\alpha^2)(1-\beta^2)}$

$= \frac{\alpha - \alpha\beta^2 + \beta - \alpha^2\beta}{1-\alpha^2-\beta^2+\alpha^2\beta^2} = \frac{(\alpha+\beta)-\alpha\beta(\alpha+\beta)}{1-(\alpha+\beta)^2+2\alpha\beta+(\alpha\beta)^2}$

$= \frac{\frac{3}{7} - \left(\frac{-2}{7}\right)\left(\frac{3}{7}\right)}{1-\left(\frac{3}{7}\right)^2+2\left(\frac{-2}{7}\right)+\left(\frac{-2}{7}\right)^2} = \frac{\frac{21+6}{49}}{\frac{49-9-28+4}{49}} = \frac{27}{16}$

3. (d) : We have, $f(x) = (3x^2 + ax - 2 - a)e^x$

$\Rightarrow f'(x) = (3x^2 + ax - 2 - a)e^x + e^x(6x + a)$

$= e^x[3x^2 + (a + 6)x - 2]$

Since, $x = 1$ is the critical point

$\therefore \quad f'(1) = 0$

$\Rightarrow 3 + (6 + a) - 2 = 0 \Rightarrow a = -7$

$\therefore \quad f'(x) = e^x[3x^2 - x - 2]$

$= e^x(3x^2 - 3x + 2x - 2)$

$= e^x(3x(x - 1) + 2(x - 1))$

$= e^x(x - 1)(3x + 2)$

+	−	+
	$\frac{-2}{3}$	1
	sign of $f'(x)$	

$\therefore \quad x = 1$ is a point of local minima and $x = \frac{-2}{3}$ is a point of local maxima.

4. (a) : We have, $(x-1)[x] \le y \le 2\sqrt{x}, 0 \le x \le 2$

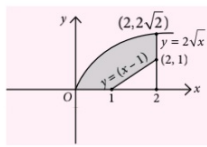
Consider $y = 2\sqrt{x} \Rightarrow y^2 = 4x, x \ge 0$

and $y = (x-1)[x] = \begin{cases} 0 & ; \quad 0 \le x \le 1 \\ x-1 & ; \quad 1 \le x < 2 \\ 2 & ; \quad x = 2 \end{cases}$

Required area

$= \int_0^2 2\sqrt{x}\,dx - \frac{1}{2} \times 1 \times 1$

$= 2\left[\frac{x^{3/2}}{3/2}\right]_0^2 - \frac{1}{2}$



$= \left(\frac{8\sqrt{2}}{3} - \frac{1}{2}\right)$ sq. units

5. (b) : Let $a, ar, ar^2, \ldots$ be the G.P. where first term $= a$ and common ratio $= r$

Now, $ar + ar^2 + ar^3 = 3$ ...(i)

Also, $ar^5 + ar^6 + ar^7 = 243$

$\Rightarrow r^4[ar + ar^2 + ar^3] = 243$

$\Rightarrow r^4(3) = 243 \quad \Rightarrow \quad r^4 = 81 = 3^4 \Rightarrow r = 3$

From (i), $3a + 9a + 27a = 3$

$\Rightarrow 39a = 3 \Rightarrow a = \frac{1}{13}$

Thus, $S_{50} = \frac{a(r^{50}-1)}{r-1} = \frac{\frac{1}{13}(3^{50}-1)}{3-1} = \frac{3^{50}-1}{26}$

6. (c) : We have, $\left(\frac{-1+i\sqrt{3}}{1-i}\right)^{30}$

$= \left(\frac{2\omega}{1-i}\right)^{30}$ $\left[\text{Since, } \omega = \frac{-1+\sqrt{3}i}{2}\right]$

$= \frac{2^{30}\cdot\omega^{30}}{((1-i)^2)^{15}} = \frac{2^{30}(\omega^3)^{10}}{(1+i^2-2i)^{15}} = \frac{2^{30}(1)}{(-2i)^{15}} = \frac{2^{30}}{(-1)^{15}2^{15}i^{15}} = -2^{15}i$

7. (d) : Given, $L = \sin^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$= \left(\frac{1-\cos\left(\frac{\pi}{8}\right)}{2}\right) - \left(\frac{1-\cos\left(\frac{\pi}{4}\right)}{2}\right)$ $\left[\because \sin^2\theta = \frac{1-\cos 2\theta}{2}\right]$

$= \frac{1}{2}\left[\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{8}\right)\right] = \frac{1}{2\sqrt{2}} - \frac{1}{2}\cos\left(\frac{\pi}{8}\right)$

Also, $M = \cos^2\left(\frac{\pi}{16}\right) - \sin^2\left(\frac{\pi}{8}\right)$

$= \left(\frac{1+\cos\left(\frac{\pi}{8}\right)}{2}\right) - \left(\frac{1-\cos\left(\frac{\pi}{4}\right)}{2}\right)$ $\left[\because \cos^2\theta = \frac{1+\cos 2\theta}{2}\right]$

$= \frac{1}{2}\left[\cos\left(\frac{\pi}{8}\right) + \cos\left(\frac{\pi}{4}\right)\right] = \frac{1}{2}\cos\left(\frac{\pi}{8}\right) + \frac{1}{2\sqrt{2}}$

8. (b) : We have, $\begin{vmatrix} x & a+y & x+a \\ y & b+y & y+b \\ z & c+y & z+c \end{vmatrix}$

$= \begin{vmatrix} x & a+y & a \\ y & b+y & b \\ z & c+y & c \end{vmatrix}$ [Applying $C_3 \to C_3 - C_1$]

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$$= \begin{vmatrix} x & y & a \\ y & y & b \\ z & y & c \end{vmatrix} \quad [\text{Applying } C_2 \to C_2 - C_3]$$

$$= \begin{vmatrix} x & y & a \\ y-x & 0 & b-a \\ z-x & 0 & c-a \end{vmatrix} \quad [\text{Applying } R_2 \to R_2 - R_1, R_3 \to R_3 - R_1]$$

$= -y\,[(y-x)(c-a) - (b-a)(z-x)]$

$= (-y)\,[(a-b)(c-a) + (a-b)(a-c-1)]$

$\left[\because x+a = y+b \Rightarrow y-x = a-b \right.$
$\left. \text{and } a+x = c+z+1 \Rightarrow z-x = a-c-1 \right]$

$= (-y)(a-b)[c-a+a-c-1] = y(a-b)$

9. (c) : Since, $y = mx + c$ is a common tangent to $\frac{x^2}{100} - \frac{y^2}{64} = 1$ and $x^2 + y^2 = 36$

Then $c^2 = 100\,m^2 - 64$ and $c^2 = 36\,m^2 + 36$

$\therefore \quad 100\,m^2 - 64 = 36\,m^2 + 36$

$\Rightarrow \quad 64\,m^2 = 100 \quad \Rightarrow \quad m^2 = \frac{100}{64} \Rightarrow m = \pm\frac{10}{8}$

$\therefore \quad c^2 = 36\left(1 + \frac{100}{64}\right) = 36 \times \frac{164}{64} = \frac{369}{4}$

$\Rightarrow \quad 4c^2 = 369$

10. (c) : We have, $x^4 e^y + 2\sqrt{y+1} = 3$

Differentiating w.r.t. x, we get

$$x^4 e^y \frac{dy}{dx} + e^y \cdot 4x^3 + \frac{2}{2\sqrt{y+1}} \frac{dy}{dx} = 0$$

At (1, 0), we have $\frac{dy}{dx} + 4 + \frac{dy}{dx} = 0 \quad \Rightarrow \quad \frac{dy}{dx} = -2$

$\therefore$ Equation of tangent at (1, 0) is

$y - 0 = -2(x - 1) \quad \Rightarrow \quad 2x + y = 2$

Point in option (c) *i.e.*, (–2, 6) only lies on it.

11. (d) :

p	q	$q \to p$	$p \to (q \to p)$	$p \vee q$	$p \to (p \vee q)$	$(p \to (q \to p)) \to (p \to (p \vee q))$
T	T	T	T	T	T	T
T	F	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T

$\therefore$ The given statement is a tautology.

12. (b) : We have, $\lim\limits_{x\to 0} x \dfrac{(e^{(\sqrt{1+x^2+x^4}-1)/x} - 1)}{\sqrt{1+x^2+x^4} - 1}$

$$= \lim_{x\to 0} \frac{(e^{(\sqrt{1+x^2+x^4}-1)/x} - 1)}{\frac{\sqrt{1+x^2+x^4}-1}{x}}$$

Put $\frac{\sqrt{1+x^2+x^4}-1}{x} = t$ As, $x \to 0$, $t \to 0$

$\therefore \quad \lim\limits_{t\to 0} \dfrac{e^t - 1}{t} \left(\dfrac{0}{0}\text{form}\right)$

Applying L' Hospital rule, we get

$$\lim_{t\to 0} \frac{e^t - 0}{1} = e^0 = 1$$

13. (a) : We have, $\log_{(7^{1/2})} x + \log_{(7^{1/3})} x + \log_{(7^{1/4})} x +$ ... 20 terms = 460

$\Rightarrow \quad \log_7 x[2 + 3 + 4 + ... + 20 + 21] = 460$

$\Rightarrow \quad \frac{20}{2}[2+21]\log_7 x = 460 \qquad \left[\because S_n = \frac{n}{2}(a+l)\right]$

$\Rightarrow \quad 230 \log_7 x = 460 \quad \Rightarrow \quad \log_7 x = 2 \quad \Rightarrow \quad x = 7^2$

14. (d) : Let $u = \tan^{-1}\left(\dfrac{\sqrt{1+x^2}-1}{x}\right)$

Putting $x = \tan\theta \quad \Rightarrow \quad \theta = \tan^{-1} x$, we get

$$u = \tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right) = \tan^{-1}\left(\frac{2\sin^2(\theta/2)}{2\sin(\theta/2)\cos(\theta/2)}\right)$$

$$= \tan^{-1}\tan(\theta/2) = \theta/2 = \frac{1}{2}\tan^{-1} x$$

Let $v = \tan^{-1}\left(\dfrac{2x\sqrt{1-x^2}}{1-2x^2}\right)$

Putting $x = \sin\theta \quad \Rightarrow \quad \theta = \sin^{-1} x$, we get

$$v = \tan^{-1}\left(\frac{2\sin\theta\cos\theta}{1-2\sin^2\theta}\right) = \tan^{-1}\left(\frac{\sin 2\theta}{\cos 2\theta}\right)$$

$= \tan^{-1}(\tan 2\theta) = 2\theta = 2\sin^{-1} x$

Now, $\dfrac{du}{dv} = \dfrac{\frac{d}{dx}(u)}{\frac{d}{dx}(v)} = \dfrac{\frac{1}{1+x^2}\cdot\frac{1}{2}}{\frac{1}{2\sqrt{1-x^2}}} = \dfrac{\sqrt{1-x^2}}{4(1+x^2)}$

$\therefore \quad$ At $x = \dfrac{1}{2}, \dfrac{du}{dv} = \dfrac{\sqrt{1-\frac{1}{4}}}{4(1+1/4)} = \dfrac{\sqrt{3}}{10}$

15. (d) : Let $I = \int \dfrac{\cos\theta}{5 + 7\sin\theta - 2\cos^2\theta} d\theta$

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$$= \int \frac{\cos\theta d\theta}{3+7\sin\theta+2\sin^2\theta}$$

Put $\sin\theta = t \Rightarrow \cos\theta\, d\theta = dt$

$$\therefore \quad I = \int \frac{dt}{2t^2+7t+3} = \int \frac{dt}{(2t+1)(t+3)}$$

$$= \int\left(\frac{2}{5(2t+1)} - \frac{1}{5(t+3)}\right)dt = \frac{1}{5}\int \frac{2}{2t+1}dt - \frac{1}{5}\int \frac{1}{t+3}dt$$

$$= \frac{1}{5}\log|2t+1| - \frac{1}{5}\log|t+3| + C = \frac{1}{5}\log\left|\frac{2t+1}{t+3}\right| + C$$

$$= \frac{1}{5}\log\left|\frac{2\sin\theta+1}{\sin\theta+3}\right| + C = A\log_e|B(\theta)| + C$$

$$\therefore \quad A = \frac{1}{5}, B(\theta) = \frac{2\sin\theta+1}{\sin\theta+3}$$

Hence, $\frac{B(\theta)}{A} = \frac{5(2\sin\theta+1)}{\sin\theta+3}$

16. (c) : We have, $\cos x\frac{dy}{dx} + 2y\sin x = \sin 2x$

$$\Rightarrow \frac{dy}{dx} + 2\tan x \cdot y = 2\sin x, \text{ which is a linear D.E.}$$

$$\therefore \quad \text{I.F.} = e^{\int P dx} = e^{\int 2\tan x\, dx} = e^{2\log\sec x} = \sec^2 x$$

The solution is given by

$$y\cdot\sec^2 x = \int 2\sin x\cdot\sec^2 x\, dx = 2\int \tan x\sec x\, dx = 2\sec x + c$$

At $x = \frac{\pi}{3}, y = 0$, we have

$$0 = 2\sec\frac{\pi}{3} + c \Rightarrow c = -4$$

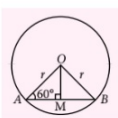
$$\therefore \quad y\sec^2 x = 2\sec x - 4$$

$$\therefore \quad \text{At } x = \frac{\pi}{4}, \text{ we have } y\cdot 2 = 2\sqrt{2} - 4 \Rightarrow y = \sqrt{2} - 2$$

17. (d) : Let the line $y - 2x = 3$ is the chord AB of the circle $x^2 + y^2 = r^2$.

Given, chord $AB = r \therefore AM = \frac{r}{2}$

ΔOAB is an equilateral triangle.



$$\therefore \quad \frac{OM}{OA} = \sin 60° \quad \Rightarrow \quad OM = \frac{\sqrt{3}}{2}r$$

Now, perpendicular distance of line AB from $O(0, 0) = OM$

$$\Rightarrow \frac{|0+0-3|}{\sqrt{1^2+2^2}} = \frac{\sqrt{3}}{2}r$$

$$\Rightarrow \frac{3}{\sqrt{5}} = \frac{\sqrt{3}}{2}r \Rightarrow r = \frac{6}{\sqrt{3}\sqrt{5}} \Rightarrow r^2 = \frac{12}{5}$$

18. (c) : Given, Mean $(\bar{x}) = 5$

$$\Rightarrow \frac{3+5+7+a+b}{5} = 5 \Rightarrow a+b = 10 \qquad \text{...(i)}$$

Also, S.D. $(\sigma) = 2$

$$\Rightarrow \sqrt{\frac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{5}} = 2$$

$$\Rightarrow (3-5)^2 + (5-5)^2 + (7-5)^2 + (a-5)^2 + (b-5)^2 = 20$$

$$\Rightarrow 4 + 0 + 4 + a^2 + 25 - 10a + b^2 + 25 - 10b = 20$$

$$\Rightarrow a^2 + b^2 - 10(a+b) + 50 + 8 = 20$$

$$\Rightarrow (a+b)^2 - 2ab - 10\times 10 + 50 + 8 = 20$$

$$\Rightarrow 10\times 10 - 2ab - 10\times 10 + 58 = 20$$

$$\Rightarrow 2ab = 38 \quad \Rightarrow \quad ab = 19 \qquad \text{...(ii)}$$

From (i) and (ii), we can get the required equation as $x^2 - 10x + 19 = 0$.

19. (b) : Given, lines $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$ and

$L_2 : \frac{x+2}{\alpha} = \frac{y+1}{5-\alpha} = \frac{z+1}{1}$ are coplanar.

$$\therefore \begin{vmatrix} 2 & -1 & 1 \\ \alpha & 5-\alpha & 1 \\ 1 & 3 & 2 \end{vmatrix} = 0$$

$$\Rightarrow 2(10 - 2\alpha - 3) + 1(2\alpha - 1) + 1(3\alpha - 5 + \alpha) = 0$$

$$\Rightarrow 14 - 4\alpha + 2\alpha - 1 + 4\alpha - 5 = 0 \quad \Rightarrow \quad 2\alpha + 8 = 0$$

$$\Rightarrow \alpha = -4$$

so, the line L_2 becomes

$$\frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$$

Point in option (b) only *i.e.*, (2, – 10, –2) lies on L_2.

20. (d) : Section A (5) Section B (5) Section C (5)

1	2	2	Selection of questions
2	1	2	
2	2	1	
1	1	3	
1	3	1	
3	1	1	

$$\therefore \quad \text{Total number of ways} = 3({}^5C_1{}^5C_2{}^5C_2) + 3({}^5C_1{}^5C_1{}^5C_3)$$

$$= 3(10\times 10\times 5) + 3(5\times 5\times 10)$$

$$= 1500 + 750 = 2250$$

21. (120) : We have, $(1 + x + x^2 + x^3)^6 = ((1+x)(1+x^2))^6$

$$= (1+x)^6(1+x^2)^6$$

$$= \sum_{r=0}^{6} {}^6C_r x^r \sum_{t=0}^{6} {}^6C_t x^{2t} = \sum_{r=0}^{6}\sum_{t=0}^{6} {}^6C_r {}^6C_t x^{r+2t}$$

For coefficient of x^4, $r + 2t = 4$

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Possibilities are $r = 0, t = 2$ or $r = 2, t = 1$, or $r = 4, t = 0$

$\therefore$ Coefficient of $x^4 = {}^6C_0\,{}^6C_2 + {}^6C_2\,{}^6C_1 + {}^6C_4\,{}^6C_0$

$= 1 \times 15 + 15 \times 6 + 15 \times 1 = 120$

22. (11) : Probability of hitting the target, $p = \frac{1}{2}$

Probability of non-hitting the target, $q = 1 - \frac{1}{2} = \frac{1}{2}$

Let n bombs are required to destroy the target.

According to question, we have

$$1 - ({}^nC_0\, p^0 q^n + {}^nC_1\, p^1 q^{n-1}) \geq \frac{99}{100}$$

$$\Rightarrow \quad 1 - \frac{1}{2^n} - \frac{n}{2^n} \geq \frac{99}{100} \Rightarrow \frac{1}{100} \geq \frac{n+1}{2^n}$$

$\Rightarrow \quad 2^n \geq (n + 1)\ 100$

At $n = 10$, $2^{10} \geq 1100$; not possible

At $n = 11$, $2^{11} \geq 1200$ which is true.

23. (0.5) : Given, $y = x^2 - 3x + 2$

At x-axis, $y = 0$

$\therefore \quad x^2 - 3x + 2 = 0 \quad \Rightarrow \quad (x - 1)(x - 2) = 0$

$\Rightarrow \quad x = 1, 2$

Now, $\frac{dy}{dx} = 2x - 3$

At $x = 1, \frac{dy}{dx} = -1$; At $x = 2, \frac{dy}{dx} = 1$

Now, $x + y = a \Rightarrow 1 + \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -1$

$\therefore$ (1, 0) lies on it.

$\Rightarrow \quad 1 + 0 = a \quad \Rightarrow \quad a = 1$

Also, $x - y = b \Rightarrow 1 - \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = 1$

$\therefore$ (2, 0) lies on it.

$\Rightarrow \quad 2 - 0 = b \Rightarrow \quad b = 2$

Hence, $\frac{a}{b} = \frac{1}{2} = 0.5$

24. (6) : We have,

Projection of $\vec{b}$ on $\vec{a}$ = Projection of $\vec{c}$ on $\vec{a}$

$$\Rightarrow \quad \frac{\vec{b}\cdot\vec{a}}{|\vec{a}|} = \frac{\vec{c}\cdot\vec{a}}{|\vec{a}|} \Rightarrow \vec{b}\cdot\vec{a} = \vec{c}\cdot\vec{a} \qquad \text{...(i)}$$

$\because \quad \vec{b}$ is perpendicular to $\vec{c}$.

$\therefore \quad \vec{b}\cdot\vec{c} = 0 \qquad \text{...(ii)}$

Let $|\vec{a} + \vec{b} - \vec{c}| = k \quad \Rightarrow \quad |\vec{a} + \vec{b} - \vec{c}|^2 = k^2$

$$\Rightarrow \quad \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}\cdot\vec{b} - 2\vec{a}\cdot\vec{c} - 2\vec{b}\cdot\vec{c} = k^2$$

$$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = k^2 \qquad \text{[Using (i) and (ii)]}$$

$\Rightarrow \quad 4 + 16 + 16 = k^2 \quad \Rightarrow \quad 36 = k^2 \quad \Rightarrow \quad k = 6$

25. (19) : We have, $C = \{f : A \to B \mid 2 \in f(A)$ and f is not one-one$\}$

Case-I : If $f(x) = 2\ \forall\ x \in A$, then number of many-one functions = 1

Case–II : If $f(x) = 2$ for exactly two elements, then total number of many-one functions $= {}^3C_2 \cdot {}^3C_1 = 9$

Case–III : If $f(x) = 2$ for exactly one element, then total number of many-one functions $= {}^3C_1 \cdot {}^3C_2 = 9$

Total number of functions = 1 + 9 + 9 = 19

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SOLVED PAPER **2020**

Karnataka CET

1. The sine of the angle between the straight line $\frac{x-2}{3}=\frac{3-y}{-4}=\frac{z-4}{5}$ and the plane $2x-2y+z=5$ is

(a) $\frac{3}{50}$ (b) $\frac{4}{5\sqrt{2}}$ (c) $\frac{\sqrt{2}}{10}$ (d) $\frac{3}{\sqrt{50}}$

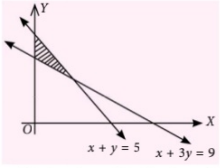
2. If a line makes an angle of $\frac{\pi}{3}$ with each of x and y-axis, then the acute angle made by z-axis is

(a) $\frac{\pi}{6}$ (b) $\frac{\pi}{3}$ (c) $\frac{\pi}{2}$ (d) $\frac{\pi}{4}$

3. The distance of the point (1, 2, −4) from the line $\frac{x-3}{2}=\frac{y-3}{3}=\frac{z+5}{6}$ is

(a) $\frac{\sqrt{293}}{7}$ (b) $\frac{293}{49}$ (c) $\frac{\sqrt{293}}{49}$ (d) $\frac{293}{7}$

4. The feasible region of an LPP is shown in the figure. If $Z=11x+7y$, then the maximum value of Z occurs at



(a) (3, 3) (b) (5, 0) (c) (3, 2) (d) (0, 5)

5. Corner points of the feasible region determined by the system of linear constraints are (0, 3), (1, 1) and (3, 0). Let $z=px+qy$, where $p, q>0$. Condition on p and q so that the minimum of z occurs at (3, 0) and (1, 1) is

(a) $p=\frac{q}{2}$ (b) $p=3q$

(c) $p=q$ (d) $p=2q$

6. If A and B are two events such that $P(A)=\frac{1}{3}$, $P(B)=\frac{1}{2}$ and $P(A\cap B)=\frac{1}{6}$, then $P\left(\frac{A'}{B}\right)$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{1}{12}$ (d) $\frac{2}{3}$

7. A die is thrown 10 times, the probability that an odd number will come up atleast one time is

(a) $\frac{1023}{1024}$ (b) $\frac{11}{1024}$ (c) $\frac{1013}{1024}$ (d) $\frac{1}{1024}$

8. The probability of solving a problem by three persons A, B and C independently is $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{3}$ respectively. Then the probability of the problem is solved by any two of them is

(a) $\frac{1}{4}$ (b) $\frac{1}{24}$ (c) $\frac{1}{8}$ (d) $\frac{1}{12}$

9. Events E_1 and E_2 form a partition of the sample space S. A is any event such that $P(E_1)=P(E_2)=\frac{1}{2}$, $P(E_2|A)=\frac{1}{2}$ and $P(A|E_2)=\frac{2}{3}$, then $P(E_1|A)$ is

(a) $\frac{2}{3}$ (b) 1 (c) $\frac{1}{4}$ (d) $\frac{1}{2}$

10. The value of $\sin^2 51° + \sin^2 39°$ is

(a) 0 (b) $\sin 12°$

(c) $\cos 12°$ (d) 1

11. If $\tan A + \cot A = 2$, then the value of $\tan^4 A + \cot^4 A =$

(a) 1 (b) 4 (c) 5 (d) 2

12. If $A = \{1, 2, 3, 4, 5, 6\}$, then the number of subsets of A which contains atleast two elements is

(a) 63 (b) 57 (c) 58 (d) 64

13. If $n(A) = 2$ and total number of possible relations from set A to set B is 1024, then $n(B)$ is

(a) 20 (b) 10 (c) 5 (d) 512

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14. The value of ${}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7$ is

(a) 1 (b) ${}^{17}C_{10}$ (c) ${}^{17}C_3$ (d) 0

15. The number of terms in the expansion of $(x + y + z)^{10}$ is

(a) 142 (b) 11 (c) 110 (d) 66

16. If $P(n) : 2^n < n!$, then the smallest positive integer for which $P(n)$ is true, is

(a) 3 (b) 4 (c) 5 (d) 2

17. If $z = x + iy$, then the equation $|z + 1| = |z - 1|$ represents

(a) a parabola (b) x-axis
(c) y-axis (d) a circle

18. If the parabola $x^2 = 4ay$ passes through the point (2, 1), then the length of the latus rectum is

(a) 4 (b) 2 (c) 8 (d) 1

19. If the sum of n terms of an A.P. is given by $S_n = n^2 + n$, then the common difference of the A.P. is

(a) 1 (b) 2 (c) 6 (d) 4

20. The two lines $lx + my = n$ and $l'x + m'y = n'$ are perpendicular if

(a) $lm' = ml'$ (b) $lm + l'm' = 0$
(c) $lm' + ml' = 0$ (d) $ll' + mm' = 0$

21. The standard deviation of the data 6, 7, 8, 9, 10 is

(a) $\sqrt{10}$ (b) 2 (c) 10 (d) $\sqrt{2}$

22. $\lim\limits_{x\to 0}\left(\dfrac{\tan x}{\sqrt{2x+4}-2}\right)$ is equal to

(a) 3 (b) 4 (c) 6 (d) 2

23. The negation of the statement "For all real numbers x and y, $x + y = y + x$" is

(a) for some real numbers x and y, $x + y = y + x$
(b) for some real numbers x and y, $x + y \neq y + x$
(c) for some real numbers x and y, $x - y = y - x$
(d) for all real numbers x and y, $x + y \neq y + x$

24. Let $f : [2, \infty) \to R$ be the function defined by $f(x) = x^2 - 4x + 5$, then the range of f is

(a) $[1, \infty)$ (b) $(1, \infty)$
(c) $[5, \infty)$ (d) $(-\infty, \infty)$

25. If A, B, C are three mutually exclusive and exhaustive events of an experiment such that $P(A) = 2P(B) = 3P(C)$, then $P(B)$ is equal to

(a) $\dfrac{2}{11}$ (b) $\dfrac{3}{11}$ (c) $\dfrac{4}{11}$ (d) $\dfrac{1}{11}$

26. If a relation R on the set {1, 2, 3} be defined by $R = \{(1, 1)\}$, then R is

(a) Reflexive and transitive
(b) Symmetric and transitive
(c) Only symmetric
(d) Reflexive and symmetric

27. The value of $\cos\left(\sin^{-1}\dfrac{\pi}{3} + \cos^{-1}\dfrac{\pi}{3}\right)$

(a) 1 (b) -1
(c) Does not exist (d) 0

28. If $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$, then A^4 is equal to

(a) $2A$ (b) I (c) $4A$ (d) A

29. If $A = \{a, b, c\}$, then the number of binary operations on A is

(a) 3^6 (b) 3^3 (c) 3^9 (d) 3

30. The domain of the function defined by $f(x) = \cos^{-1}\sqrt{x-1}$ is

(a) [0, 2] (b) [−1, 1] (c) [0, 1] (d) [1, 2]

31. If $f(x) = \begin{vmatrix} x^3 - x & a + x & b + x \\ x - a & x^2 - x & c + x \\ x - b & x - c & 0 \end{vmatrix}$, then

(a) $f(2) = 0$ (b) $f(0) = 0$
(c) $f(-1) = 0$ (d) $f(1) = 0$

32. If A and B are square matrices of same order and B is a skew symmetric matrix, then $A'BA$ is

(a) Null matrix (b) Diagonal matrix
(c) Skew symmetric matrix
(d) Symmetric matrix

33. If $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then the matrix A is

(a) $\begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$ (b) $\begin{pmatrix} -2 & 1 \\ 3 & -2 \end{pmatrix}$

(c) $\begin{pmatrix} 2 & -1 \\ 3 & 2 \end{pmatrix}$ (d) $\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$

34. If $f(x) = \begin{cases} \dfrac{1 - \cos Kx}{x \sin x}, & \text{if } x \neq 0 \\ \dfrac{1}{2}, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then the value of K is

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(a) 0 (b) ±2 (c) ±1 (d) $\pm\frac{1}{2}$

35. If $a_1, a_2, a_3, \ldots\ldots\ldots, a_9$ are in A.P., then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

(a) $a_1 + a_9$ (b) $\log_e(\log_e e)$

(c) 1 (d) $\frac{9}{2}(a_1 + a_9)$

36. If A is a square matrix of order 3 and $|A| = 5$, then $|A \text{ adj } A|$ is

(a) 125 (b) 25 (c) 625 (d) 5

37. If $f(x) = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$, then $f'(\sqrt{3})$ is

(a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $-\frac{1}{\sqrt{3}}$ (d) $-\frac{1}{2}$

38. The right hand and left hand limit of the function

$$f(x) = \begin{cases} \dfrac{e^{1/x} - 1}{e^{1/x} + 1}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$$ are respectively

(a) 1 and –1 (b) –1 and –1

(c) –1 and 1 (d) 1 and 1

39. If $2^x + 2^y = 2^{x+y}$, then $\frac{dy}{dx}$ is

(a) -2^{y-x} (b) 2^{x-y}

(c) $\frac{2^y - 1}{2^x - 1}$ (d) 2^{y-x}

40. If the curves $2x = y^2$ and $2xy = K$ intersect perpendicularly, then the value of K^2 is

(a) $2\sqrt{2}$ (b) 2 (c) 8 (d) 4

41. If $(xe)^y = e^x$, then $\frac{dy}{dx}$ is

(a) $\frac{1}{(1+\log x)^2}$ (b) $\frac{\log x}{(1+\log x)}$

(c) $\frac{e^x}{x(y-1)}$ (d) $\frac{\log x}{(1+\log x)^2}$

42. If $y = 2x^{n+1} + \frac{3}{x^n}$, then $x^2\frac{d^2y}{dx^2}$ is

(a) $n(n+1)y$ (b) $x\frac{dy}{dx} + y$

(c) y (d) $6n(n+1)y$

43. The value of $\int \frac{1+x^4}{1+x^6}\,dx$ is

(a) $\tan^{-1}x + \frac{1}{3}\tan^{-1}x^3 + C$

(b) $\tan^{-1}x - \frac{1}{3}\tan^{-1}x^3 + C$

(c) $\tan^{-1}x + \frac{1}{3}\tan^{-1}x^2 + C$

(d) $\tan^{-1}x + \tan^{-1}x^3 + C$

44. The maximum value of $\frac{\log_e x}{x}$, if $x > 0$ is

(a) 1 (b) $\frac{1}{e}$ (c) $-\frac{1}{e}$ (d) e

45. If the side of a cube is increased by 5%, then the surface area of a cube is increased by

(a) 60% (b) 6% (c) 20% (d) 10%

46. The value of $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos^{-1}x\,dx$ is

(a) $\frac{\pi}{2}$ (b) 1 (c) $\frac{\pi^2}{2}$ (d) π

47. If $\int \frac{3x+1}{(x-1)(x-2)(x-3)}dx = A\log|x-1| + B\log|x-2| + C\log|x-3| + c$, then the values of A, B and C are respectively,

(a) 2, –7, –5 (b) 5, –7, 5

(c) 2, –7, 5 (d) 5, –7, –5

48. The value of $\int e^{\sin x}\sin 2x\,dx$ is

(a) $2e^{\sin x}(\sin x + 1) + C$

(b) $2e^{\sin x}(\cos x + 1) + C$

(c) $2e^{\sin x}(\cos x - 1) + C$

(d) $2e^{\sin x}(\sin x - 1) + C$

49. The area of the region bounded by the curve $y^2 = 8x$ and the line $y = 2x$ is

(a) $\frac{4}{3}$ sq. units (b) $\frac{3}{4}$ sq. units

(c) $\frac{8}{3}$ sq. units (d) $\frac{16}{3}$ sq. units

50. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\cos x}{1+e^x}\,dx$ is

(a) 0 (b) 1 (c) –2 (d) 2

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51. The value of $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$ is

(a) $\frac{\pi}{4}\log 2$ (b) $\frac{1}{2}$

(c) $\frac{\pi}{8}\log 2$ (d) $\frac{\pi}{2}\log 2$

52. The general solution of the differential equation $x^2dy - 2xydx = x^4\cos x\,dx$ is

(a) $y = x^2\sin x + c$ (b) $y = \sin x + cx^2$

(c) $y = \cos x + cx^2$ (d) $y = x^2\sin x + cx^2$

53. The area of the region bounded by the line $y = 2x + 1$, x-axis and the ordinates $x = -1$ and $x = 1$ is

(a) 2 (b) 5/2 (c) 5 (d) 9/4

54. The order of the differential equation obtained by eliminating arbitrary constants in the family of curves $c_1y = (c_2 + c_3)e^{x+c_4}$ is

(a) 2 (b) 3 (c) 4 (d) 1

55. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ is the angle between $\vec{a}$ and $\vec{b}$, then $\sin\frac{\theta}{2}$ is

(a) $\frac{|\vec{a}+\vec{b}|}{2}$ (b) $\frac{|\vec{a}-\vec{b}|}{2}$

(c) $|\vec{a}-\vec{b}|$ (d) $|\vec{a}+\vec{b}|$

56. The curve passing through the point (1, 2) given that the slope of the tangent at any point (x, y) is $\frac{2x}{y}$ represents

(a) Parabola (b) Ellipse

(c) Hyperbola (d) Circle

57. The two vectors $\hat{i}+\hat{j}+\hat{k}$ and $\hat{i}+3\hat{j}+5\hat{k}$ represents the two sides $\overline{AB}$ and $\overline{AC}$ respectively of a ΔABC. The length of the median though A is

(a) 14 (b) 7 (c) $\sqrt{14}$ (d) $\frac{\sqrt{14}}{2}$

58. The point (1, –3, 4) lies in the octant

(a) Third (b) Fourth

(c) Eighth (d) Second

59. If the vectors $2\hat{i}-3\hat{j}+4\hat{k}, 2\hat{i}+\hat{j}-\hat{k}$ and $\lambda\hat{i}-\hat{j}+2\hat{k}$ are coplanar, then the value of λ is

(a) –5 (b) –6 (c) 5 (d) 6

60. If $|\vec{a}\times\vec{b}|^2 + |\vec{a}\cdot\vec{b}|^2 = 144$ and $|\vec{a}| = 6$, then $|\vec{b}|$ is equal to

(a) 3 (b) 2 (c) 4 (d) 6

SOLUTIONS

1. (c) : The given equations of line and plane are

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$

and $2x - 2y + z - 5 = 0$

Now, let θ be the angle between them.

$$\therefore \sin\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2+b_1^2+c_1^2}\sqrt{a_2^2+b_2^2+c_2^2}}$$

$$= \frac{3\cdot 2 + 4\cdot(-2) + 5(1)}{\sqrt{3^2+4^2+5^2}\sqrt{2^2+(-2)^2+(1)^2}} = \frac{6-8+5}{5\sqrt{2}\cdot 3}$$

$$= \frac{1}{5\sqrt{2}} = \frac{\sqrt{2}}{10}$$

2. (d) : Given, $\alpha = \frac{\pi}{3}, \beta = \frac{\pi}{3}$

Let γ be the angle made by the line with z-axis.

We know that, $l^2 + m^2 + n^2 = 1$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 + \cos^2\gamma = 1$

$\Rightarrow \cos^2\gamma = 1 - \frac{1}{2} = \frac{1}{2} \Rightarrow \cos\gamma = \pm\frac{1}{\sqrt{2}}$

$\Rightarrow \gamma = \frac{\pi}{4}$ ($\because \gamma$ is acute)

3. (a) : The given equation of line is,

$$\frac{x-3}{2} = \frac{y-3}{3} = \frac{z-(-5)}{6} = \lambda \text{ (say)}$$

$\therefore$ Any point on the line is $P(2\lambda + 3, 3\lambda + 3, 6\lambda - 5)$

Now, d.r.'s of the line joining the point $P(2\lambda + 3, 3\lambda + 3, 6\lambda - 5)$ and $Q(1, 2, -4)$ are

$<2\lambda + 3 - 1, 3\lambda + 3 - 2, 6\lambda - 5 + 4>$

i.e., $<2\lambda + 2, 3\lambda + 1, 6\lambda - 1>$

Now, line PQ is perpendicular to the given line.

$\therefore 2(2\lambda + 2) + 3(3\lambda + 1) + 6(6\lambda - 1) = 0$

$\Rightarrow 4\lambda + 4 + 9\lambda + 3 + 36\lambda - 6 = 0$

$\Rightarrow 49\lambda + 1 = 0$

$\Rightarrow \lambda = -1/49$

$\therefore$ Coordinates of P are

$$\left(2\left(\frac{-1}{49}\right) + 3, 3\left(\frac{-1}{49}\right) + 3, 6\left(\frac{-1}{49}\right) - 5\right)$$

i.e., $\left(\frac{145}{49}, \frac{144}{49}, \frac{-251}{49}\right)$

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$\therefore$ Required distance,

$$PQ=\sqrt{\left(\frac{145}{49}-1\right)^2+\left(\frac{144}{49}-2\right)^2+\left(\frac{-251}{49}+4\right)^2}$$

$$=\sqrt{\frac{9216}{49^2}+\frac{2116}{49^2}+\frac{3025}{49^2}}=\sqrt{\frac{14357}{49^2}}=\frac{\sqrt{293}}{7}\text{ units}$$

4. (c) : The given equations of lines are

$x+y=5$ ...(i)

and $x+3y=9$ ...(ii)

Their point of intersection is (3, 2).

Also, point of intersection of (i) and y-axis is (0, 5).

Similarly, point of intersection of (ii) and y-axis is (0, 3).

So, the corner point of the feasible region are $A(3, 2)$, $B(0, 5)$ and $C(0, 3)$.

Corner points	Value of $Z = 11x + 7y$
$A(3, 2)$	$11 \times 3 + 7 \times 2 = 47$ (Maximum)
$B(0, 5)$	$11 \times 0 + 7 \times 5 = 35$
$C(0, 3)$	$11 \times 0 + 7 \times 3 = 21$

So, maximum value of Z occurs at (3, 2).

5. (a) : We are given that, $z = px + qy$

Also, minimum value of z occurs at (3, 0) and (1, 1)

$\Rightarrow$ z at (3, 0) = z at (1, 1)

$\Rightarrow \quad 3p + 0 = p + q \Rightarrow 2p = q \Rightarrow p = \frac{q}{2}$

6. (d) : We have,

$$P(A)=\frac{1}{3},\ P(B)=\frac{1}{2}\text{ and }P(A\cap B)=\frac{1}{6}$$

$$P(A'/B)=\frac{P(A'\cap B)}{P(B)}=\frac{P(B)-P(A\cap B)}{P(B)}$$

$$=1-\frac{P(A\cap B)}{P(B)}=1-\frac{1/6}{1/2}=1-\frac{1}{3}=\frac{2}{3}$$

7. (a) : We have, probability of odd number $=\frac{1}{2}=p$

$\therefore \quad q=\frac{1}{2}$

Now, $n = 10$

$\therefore$ Probability that an odd number comes up atleast one time = 1 – probability that no odd number comes up

$$=1-{}^nC_0\,p^0\,q^n$$

$$=1-{}^{10}C_0\left(\frac{1}{2}\right)^{10}=1-\frac{1}{2^{10}}=\frac{1023}{1024}$$

8. (a) : We have, $P(A)=\frac{1}{2},\ P(B)=\frac{1}{4}$ and $P(C)=\frac{1}{3}$

Now, required probability = $P(A \cap B \cap C')$

$+ P(A \cap B' \cap C) + P(A' \cap B \cap C)$

$= P(A)\cdot P(B)\cdot P(C') + P(A)\cdot P(B')\cdot P(C) + P(A')\cdot P(B)\cdot P(C)$

($\because$ A, B, C are independent events)

$$=\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{2}{3}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{3}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{3}=\frac{2+3+1}{24}=\frac{1}{4}$$

9. (d) : We have, $P(E_1)=P(E_2)=\frac{1}{2}$,

$P(E_2/A)=\frac{1}{2},\ P(A/E_2)=\frac{2}{3}$

Let $P\left(\frac{A}{E_1}\right)$ be p.

Using Bayes' theorem, we have

$$P(E_2/A)=\frac{P(A/E_2)\cdot P(E_2)}{P(A/E_1)\cdot P(E_1)+P(A/E_2)\cdot P(E_2)}$$

$$\Rightarrow \quad \frac{1}{2}=\frac{\frac{2}{3}\cdot\frac{1}{2}}{p\cdot\frac{1}{2}+\frac{2}{3}\cdot\frac{1}{2}} \quad \Rightarrow \quad p+\frac{2}{3}=\frac{2}{3}\times 2$$

$$\Rightarrow \quad p=\frac{2}{3} \Rightarrow P(A/E_1)=\frac{2}{3}$$

Now, $P(E_1/A)=\frac{P(A/E_1)\cdot P(E_1)}{P(A/E_1)\cdot P(E_1)+P(A/E_2)\cdot P(E_2)}$

$$=\frac{\frac{2}{3}\cdot\frac{1}{2}}{\frac{2}{3}\cdot\frac{1}{2}+\frac{2}{3}\cdot\frac{1}{2}}=\frac{1}{2}$$

10. (d) : We have, $\sin^2 51° + \sin^2 39°$

$= \sin^2 51° + \sin^2(90° - 51°) = \sin^2 51° + \cos^2 51° = 1$

11. (d) : We have, $\tan A + \cot A = 2$

Squaring both sides, we get

$\tan^2 A + \cot^2 A + 2 = 4 \qquad \left(\because \tan A = \frac{1}{\cot A}\right)$

$\Rightarrow \quad \tan^2 A + \cot^2 A = 2$

Again squaring both sides, we get

$\tan^4 A + \cot^4 A + 2\tan^2 A\cdot\cot^2 A = 4$

$\Rightarrow \quad \tan^4 A + \cot^4 A = 2$

12. (b) : We have, $n(A) = 6$

Total number of subsets of $A = 2^6$

$\therefore$ Number of subsets of A which contains atleast two elements = $2^6 - 7$

[$\because$ Subsets of A having none or 1 element are $\{\phi\}, \{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}$]

$= 64 - 7 = 57$

13. (c) : We have, $n(A) = 2$

Let number of elements in set B be p.

Then, number of possible relations from A to $B = 2^{2p}$

$\Rightarrow \quad 1024 = 2^{2p} \Rightarrow 2^{10} = 2^{2p} \Rightarrow p = 5 \Rightarrow n(B) = 5$

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14. (d) : We have, ${}^{16}C_9 + {}^{16}C_{10} - {}^{16}C_6 - {}^{16}C_7$

$= {}^{17}C_{10} - {}^{17}C_7 \qquad (\because {}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r)$

$= {}^{17}C_{10} - {}^{17}C_{10} = 0 \qquad (\because {}^nC_r = {}^nC_{n-r})$

15. (d) : Required number of terms

$$= {}^{10+3-1}C_{10} = {}^{12}C_{10} = \frac{12!}{10!\,2!} = 66$$

16. (b) : We have, $P(n) : 2^n < n!$

For $n = 1$, we get

$2^1 < 1!$ (false)

For $n = 2$, we get

$2^2 < 2!$ (false)

For $n = 3$, we get

$2^3 < 3!$ (false)

For $n = 4$, we get

$2^4 < 4!$ (true)

So, the smallest positive integer for which $P(n)$ is true is 4.

17. (c) : We have, $z = x + iy$

Also, $|z + 1| = |z - 1|$

$\Rightarrow |(x + 1) + (iy)| = |(x - 1) + iy|$

$\Rightarrow (x + 1)^2 + y^2 = (x - 1)^2 + y^2$

$\Rightarrow x^2 + 1 + 2x = x^2 + 1 - 2x$

$\Rightarrow 4x = 0 \Rightarrow x = 0$

i.e., y-axis.

18. (a) : The given parabola passes through (2, 1).

$\therefore \quad (2)^2 = 4 \cdot a \cdot (1) \Rightarrow a = 1$

So, length of latus rectum $= 4a = 4(1) = 4$

19. (b) : We have, $S_n = n^2 + n$

$\Rightarrow S_{n-1} = (n - 1)^2 + (n - 1) = (n - 1)[n - 1 + 1]$

$= n(n - 1) = n^2 - n$

$\therefore \quad T_n = S_n - S_{n-1} = n^2 + n - (n^2 - n) = 2n$

$\Rightarrow T_{n-1} = 2(n - 1) = 2n - 2$

$\therefore$ Common difference $= T_n - T_{n-1} = 2n - 2n + 2 = 2$

20. (d) : Since the given lines are perpendicular.

$\therefore$ Product of their slopes $= -1$

$$\Rightarrow \quad \frac{-l}{m} \times \frac{-l'}{m'} = -1 \Rightarrow ll' = -mm' \Rightarrow ll' + mm' = 0$$

21. (d) : Mean $(\bar{x}) = \dfrac{6 + 7 + 8 + 9 + 10}{5} = 8$

Now, $\Sigma x_i^2 = 6^2 + 7^2 + 8^2 + 9^2 + 10^2 = 330$

$$\therefore \quad \text{Standard deviation, } \sigma = \sqrt{\frac{\sum x_i^2}{n} - (\bar{x})^2}$$

$$= \sqrt{\frac{330}{5} - (8)^2} = \sqrt{2}$$

22. (d) : We have, $\lim\limits_{x \to 0} \dfrac{\tan x}{\sqrt{2x + 4} - 2}$ $\left(\dfrac{0}{0} \text{ form}\right)$

Applying L' Hospital's Rule, we get

$$\lim_{x \to 0} \frac{\sec^2 x}{\frac{1}{2} \times \frac{2}{\sqrt{2x + 4}}} = \lim_{x \to 0} \left(\sqrt{2x + 4} \cdot \sec^2 x\right) = 2 \cdot 1 = 2$$

23. (b) : The negation of the given statement is "For some real numbers x and y, $x + y \neq y + x$".

24. (a) : Let $y = x^2 - 4x + 5$

$\Rightarrow \quad y - 1 = (x - 2)^2$

$\Rightarrow \quad x - 2 = \sqrt{y - 1} \Rightarrow x = 2 + \sqrt{y - 1}$

$\therefore \quad y - 1 \geq 0 \Rightarrow y \geq 1$

So, range of $f(x) = [1, \infty)$

25. (b) : We have, A, B, C are mutually exclusive and exhaustive events.

$\therefore \quad A \cap B = B \cap C = C \cap A = A \cap B \cap C = \phi$

$\Rightarrow \quad P(A \cap B) = P(B \cap C) = P(C \cap A) = P(A \cap B \cap C) = 0$

Now, $P(A) = 2P(B) = 3P(C)$

$$\Rightarrow \quad P(A) = 2P(B) \text{ and } P(C) = \frac{2}{3}P(B)$$

$\therefore \quad P(A \cup B \cup C) = P(A) + P(B) + P(C)$

$$\Rightarrow \quad 1 = 2P(B) + P(B) + \frac{2}{3}P(B)$$

$$\Rightarrow \quad 1 = \frac{11}{3}P(B) \Rightarrow P(B) = \frac{3}{11}$$

26. (b) : R is symmetric and transitive but not reflexive as (2, 2) and (3, 3) $\notin R$.

27. (c)

28. (b) : We have, $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \quad A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$\Rightarrow \quad A^4 = I^2 = I$

29. (c) : We have, $n(A) = 3$

So, total number of binary operations on $A = 3^{3^2} = 3^9$

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30. (d) : We have, $f(x)=\cos^{-1}\sqrt{x-1}$

We know that, $\cos^{-1}x$ will be defined if $-1\le x\le 1$

$\Rightarrow \quad -1\le\sqrt{x-1}\le 1$

$\Rightarrow \quad 0\le x-1\le 1 \Rightarrow 1\le x\le 2$

$\therefore$ Domain of $f(x)=[1,2]$

31. (b) : We have, $f(x)=\begin{vmatrix} x^3-x & a+x & b+x \\ x-a & x^2-x & c+x \\ x-b & x-c & 0 \end{vmatrix}$

$$\Rightarrow \quad f(0)=\begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix}$$

$$=0-a(0+bc)+b(ac-0)=0$$

32. (c) : We have, B is a skew symmetric matrix.

i.e., $B'=-B$

$\therefore \quad (A'BA)'=A'B'A=A'(-B)\cdot A=-A'BA$

$\therefore \quad A'BA$ is a skew symmetric matrix.

33. (a) : We have, $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}A=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \quad BA=I$ (say) $\Rightarrow A=B^{-1}$

Now, $\text{adj } B=\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}'=\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

and $|B|=4-3=1$

$\therefore \quad B^{-1}=\dfrac{(\text{adj } B)}{|B|}=\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

So, $A=\begin{bmatrix} 2 & -1 \\ -3 & 2 \end{bmatrix}$

34. (c) : We have, $f(x)$ is continuous at $x=0$.

$\therefore \quad f(0)=\lim\limits_{x\to 0}\dfrac{1-\cos Kx}{x\sin x}$ $\left(\dfrac{0}{0}\text{ form}\right)$

$\Rightarrow \quad \dfrac{1}{2}=\lim\limits_{x\to 0}\dfrac{K\sin Kx}{x\cdot\cos x+\sin x}$

(Applying L' Hospital's Rule)

$\Rightarrow \quad \dfrac{1}{2}=\lim\limits_{x\to 0}\dfrac{K^2\cdot\cos Kx}{x(-\sin x)+\cos x+\cos x}$

(Applying L' Hospital's Rule)

$\Rightarrow \quad \dfrac{1}{2}=\dfrac{K^2}{0+1+1}\Rightarrow K^2=1\Rightarrow K=\pm 1$

35. (b) : Let a be the first term and d be the common difference of the A.P. Then,

$a_1=a,\ a_2=a+d,\ \ldots\ldots,\ a_9=a+8d$

So, $\Delta=\begin{vmatrix} a & a+d & a+2d \\ a+3d & a+4d & a+5d \\ a+6d & a+7d & a+8d \end{vmatrix}$

Applying $C_2\to C_2-C_1$, $C_3\to C_3-C_1$, we get

$$\Delta=\begin{vmatrix} a & d & 2d \\ a+3d & d & 2d \\ a+6d & d & 2d \end{vmatrix}=d\cdot 2d\begin{vmatrix} a & 1 & 1 \\ a+3d & 1 & 1 \\ a+6d & 1 & 1 \end{vmatrix}$$

$=0\ (\because\ C_2\sim C_3)$

$=\log_e(\log_e e)\qquad (\because \log_e e=1 \text{ and } \log_e 1=0)$

36. (a) : We have, $|A|=5$

$\therefore \quad |A\ \text{adj}\ A|=|A|\ |\text{adj}\ (A)|=|A|^n=(5)^3=125$

37. (a) : We have, $f(x)=\sin^{-1}\left(\dfrac{2x}{1+x^2}\right)$

Put $x=\tan\theta\ \Rightarrow\ \theta=\tan^{-1}x$

$\therefore \quad f(x)=\sin^{-1}\left(\dfrac{2\tan\theta}{1+\tan^2\theta}\right)=\sin^{-1}(\sin 2\theta)=2\theta$

$\Rightarrow \quad f(x)=2\tan^{-1}x$

Differentiating w.r.t. x both sides, we get

$$f'(x)=\frac{2}{1+x^2}$$

$\Rightarrow \quad f'(\sqrt{3})=\dfrac{2}{1+3}=\dfrac{1}{2}$

38. (a) : R.H.L. $=\lim\limits_{x\to 0^+}\dfrac{e^{1/x}-1}{e^{1/x}+1}$

$$=\lim_{h\to 0}\frac{e^{1/h}-1}{e^{1/h}+1}=\lim_{h\to 0}\frac{1-e^{-1/h}}{1+e^{-1/h}}=\frac{1-0}{1+0}=1$$

L.H.L. $=\lim\limits_{x\to 0^-}\dfrac{e^{1/x}-1}{e^{1/x}+1}=\lim\limits_{h\to 0}\dfrac{e^{-1/h}-1}{e^{-1/h}+1}=\dfrac{0-1}{0+1}=-1$

39. (a) : We have, $2^x+2^y=2^{x+y}$

Dividing both sides by 2^{x+y}, we get

$2^{-y}+2^{-x}=1$

Differentiating both sides w.r.t. x, we get

$$-2^{-y}\log_e 2\cdot\frac{dy}{dx}-2^{-x}\log_e 2=0\Rightarrow \frac{dy}{dx}=-2^{y-x}$$

40. (c) : The given curves are,

$2x=y^2$ ...(i) and $2xy=K$ ...(ii)

Solving (i) and (ii), we get

$y^3=K\Rightarrow y=K^{1/3}$

$\therefore \quad x=\dfrac{K^{2/3}}{2}$

So, point of intersection of given curves is $\left(\dfrac{K^{2/3}}{2},K^{1/3}\right)$.

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Now, differentiating (i) and (ii) w.r.t. x, we get

$$2 = 2y \cdot \frac{dy}{dx} \text{ and } 2\left(x\frac{dy}{dx} + y\right) = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{y} \text{ and } \frac{dy}{dx} = \frac{-y}{x}$$

$$\Rightarrow m_1 = \frac{1}{K^{1/3}} \text{ and } m_2 = \frac{-2K^{1/3}}{K^{2/3}} = \frac{-2}{K^{1/3}}$$

$\because$ Given curves are perpendicular.

$\therefore m_1 m_2 = -1$

$$\Rightarrow \frac{1}{K^{1/3}} \cdot \left(\frac{-2}{K^{1/3}}\right) = -1$$

$$\Rightarrow 2 = K^{2/3} \Rightarrow K^2 = 2^3 = 8$$

41. (d) : We have, $(x \cdot e)^y = e^x$

Taking log on both sides, we get

$$y \log (x \cdot e) = x \log e \Rightarrow y \log x + y = x \quad \ldots(i)$$

Differentiating (i) w.r.t. x, we get

$$y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} + \frac{dy}{dx} = 1 \Rightarrow \frac{dy}{dx}(1 + \log x) = 1 - \frac{y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x - y}{x(1 + \log x)} = \frac{x - \dfrac{x}{1 + \log x}}{x(1 + \log x)} \quad \text{(Using (i))}$$

$$= \frac{\log x}{(1 + \log x)^2}$$

42. (a) : We have, $y = 2x^{n+1} + \dfrac{3}{x^n}$

$$\Rightarrow \frac{dy}{dx} = 2(n+1) \cdot x^n - \frac{3n}{x^{n+1}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2(n+1) \cdot n \cdot x^{n-1} + \frac{3n(n+1)}{x^{n+2}}$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} = 2n(n+1) \cdot x^{n+1} + \frac{3n(n+1)}{x^n}$$

$$= n(n+1)\left[2x^{n+1} + \frac{3}{x^n}\right]$$

$$\Rightarrow x^2 \frac{d^2y}{dx^2} = n(n+1)y$$

43. (a) : Let $I = \int \dfrac{1 + x^4}{1 + x^6}\, dx$

$$= \int \frac{1 + x^2 - x^2 + x^4}{(1 + x^2)(1 + x^4 - x^2)}\, dx$$

$$= \int \frac{1 + x^4 - x^2}{(1 + x^2)(1 + x^4 - x^2)}\, dx + \int \frac{x^2}{1 + x^6}\, dx$$

$$= \int \frac{1}{1 + x^2}\, dx + \frac{1}{3}\int \frac{3x^2}{1 + (x^3)^2}\, dx$$

$$= \tan^{-1} x + \frac{1}{3}\tan^{-1}(x^3) + C$$

44. (b) : Let $y = \dfrac{\log_e x}{x}$

$$\Rightarrow \frac{dy}{dx} = \frac{x \cdot \dfrac{1}{x} - \log x}{x^2} \Rightarrow \frac{dy}{dx} = \frac{1 - \log x}{x^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{x^2 \cdot \left(\dfrac{-1}{x}\right) + (1 - \log x) \cdot 2x}{x^4} = \frac{x - 2x \log x}{x^4}$$

For maxima/minima, $\dfrac{dy}{dx} = 0$

$$\Rightarrow 1 - \log x = 0 \Rightarrow x = e$$

And $\left[\dfrac{d^2y}{dx^2}\right]_{(x=e)} = \dfrac{-e}{e^4} < 0$

$\therefore$ y is maximum at $x = e$.

And maximum value of $y = \dfrac{1}{e}$.

45. (d) : Let a be the side of the cube.

$$\therefore \frac{da}{dt} = \frac{5a}{100}$$

Now, $A = 6a^2$

$$\Rightarrow \frac{dA}{dt} = 12a\left(\frac{da}{dt}\right)$$

$$= 12a \cdot \frac{5a}{100} = \frac{60}{100}a^2 = \left(\frac{10}{100}\right)A$$

$\therefore$ Surface area of cube is increased by 10%.

46. (a) : Let $I = \int_{-1/2}^{1/2} \cos^{-1} x\, dx$

$$= [\cos^{-1} x \cdot x]_{-1/2}^{1/2} + \int_{-1/2}^{1/2} \frac{x}{\sqrt{1 - x^2}}\, dx$$

$$= \frac{1}{2}\cos^{-1}\left(\frac{1}{2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{-1}{2}\right) - \left[\sqrt{1 - x^2}\right]_{-1/2}^{1/2}$$

$$= \frac{1}{2} \cdot \frac{\pi}{3} + \frac{1}{2} \cdot \frac{2\pi}{3} - 0 = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$$

47. (c) : Let $I = \int \dfrac{(3x + 1)}{(x - 1)(x - 2)(x - 3)}\, dx$

Let $\dfrac{3x + 1}{(x - 1)(x - 2)(x - 3)} = \dfrac{P}{x - 1} + \dfrac{Q}{x - 2} + \dfrac{R}{x - 3}$

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$\Rightarrow \quad 3x + 1 = P(x-2)(x-3) + Q(x-1)(x-3) + R(x-1)(x-2)$

Putting $x = 1$, we get $4 = 2P \Rightarrow P = 2$

Putting $x = 2$, we get $7 = -Q \Rightarrow Q = -7$

Putting $x = 3$, we get $10 = 2R \Rightarrow R = 5$

$$\therefore \quad I = \int \left[\frac{2}{x-1} - \frac{7}{x-2} + \frac{5}{x-3}\right] dx$$

$$= 2\log|x-1| - 7\log|x-2| + 5\log|x-3| + c$$

So, $A = 2, B = -7$ and $C = 5$

48. (d) : Let $I = \int e^{\sin x} \cdot \sin 2x \, dx = \int e^{\sin x} 2\sin x \cos x dx$

Put $e^{\sin x} = t \Rightarrow \sin x = \log t$

$\Rightarrow \quad e^{\sin x} \cos x \, dx = dt$

$$\therefore \quad I = \int 2\log t \cdot dt = 2\left[\log t \cdot t - \int \frac{1}{t} \cdot t \, dt\right]$$

$$= 2[t\log t - t] + C = 2[e^{\sin x} \cdot \sin x - e^{\sin x}] + C$$

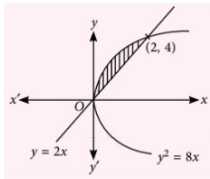
$$= 2e^{\sin x}(\sin x - 1) + C$$

49. (a) : The given curves are

$y^2 = 8x$ ...(i)

and $y = 2x$ ...(ii)

Points of intersection of (i) and (ii) are (0,0), (2, 4).



$\therefore$ Required area

$$= \int_0^2 (\sqrt{8x} - 2x)\, dx$$

$$= 2\sqrt{2}\left[\frac{2x^{3/2}}{3}\right]_0^2 - [x^2]_0^2$$

$$= \frac{4\sqrt{2}}{3} \cdot \sqrt{8} - 4 = \frac{16}{3} - 4 = \frac{4}{3} \text{ sq. units}$$

50. (b) : Let $I = \int_{-\pi/2}^{\pi/2} \frac{\cos x}{1+e^x} dx$ ...(i)

$$\Rightarrow \quad I = \int_{-\pi/2}^{\pi/2} \frac{\cos(-x)}{1+e^{-x}} dx = \int_{-\pi/2}^{\pi/2} \frac{e^x \cdot \cos x}{1+e^x} dx \quad \text{....(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_{-\pi/2}^{\pi/2} \cos x \, dx \Rightarrow I = \frac{1}{2} \cdot 2 \int_0^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2}$$

$\Rightarrow \quad I = 1$

51. (c) : Let $I = \int_0^1 \frac{\log(1+x)}{1+x^2} dx$

Put $x = \tan\theta \Rightarrow dx = \sec^2\theta \, d\theta$

When $x = 0, \theta = 0$ and when $x = 1, \theta = \frac{\pi}{4}$

$$\therefore \quad I = \int_0^{\pi/4} \log(1+\tan\theta)\, d\theta \quad \text{...(i)}$$

$$= \int_0^{\pi/4} \log\left(1 + \tan\left(\frac{\pi}{4} - \theta\right)\right) d\theta$$

$$= \int_0^{\pi/4} \log\left[1 + \frac{1-\tan\theta}{1+\tan\theta}\right] d\theta$$

$$\Rightarrow \quad I = \int_0^{\pi/4} (\log 2 - \log(1+\tan\theta))\, d\theta \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/4} \log 2 \; d\theta = \log 2 [\theta]_0^{\pi/4}$$

$$\Rightarrow \quad I = \frac{1}{2} \cdot \log 2 \cdot \frac{\pi}{4} = \frac{\pi}{8} \log 2$$

52. (d) : The given differential equation is,

$x^2\, dy - 2xy\, dx = x^4 \cos x \, dx$

$$\Rightarrow \quad \frac{dy}{dx} - \frac{2y}{x} = x^2 \cos x,$$

which is a linear differential equation.

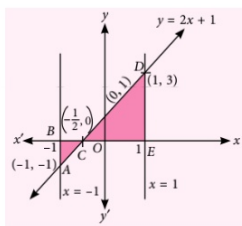
$$\therefore \quad \text{I.F.} = e^{\int -2/x\, dx} = e^{-2\log x} = \frac{1}{x^2}$$

So, required solution is,

$$\frac{y}{x^2} = \int \cos x \, dx + c \Rightarrow y = x^2 \sin x + cx^2$$

53. (b) : The given lines are,

$y = 2x + 1$, $x = -1$, $x = 1$ and x-axis



$\therefore$ Required area $= \text{ar}(\Delta ABC) + \text{ar}(\Delta CDE)$

$$= \frac{1}{2} \times \frac{1}{2} \times 1 + \frac{1}{2} \times \frac{3}{2} \times 3 = \frac{1}{4} + \frac{9}{4} = \frac{10}{4} = \frac{5}{2} \text{ sq. units}$$

54. (d) : The given differential equation is,

$c_1 y = (c_2 + c_3)\, e^{x+c_4}$

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$\Rightarrow \quad c_1 y = A \cdot e^x$, where $A = (c_2 + c_3) \cdot e^{c_4}$

$\Rightarrow \quad y = Be^x$, where $B = \dfrac{A}{c_1}$

Since, the given equation has only one arbitrary constant.

$\therefore$ The order of the differential equation is 1.

55. (b) : $\because$ $\vec{a}$ and $\vec{b}$ are unit vectors.

$\therefore \quad |\vec{a}| = |\vec{b}| = 1$

Now, $|\vec{a} - \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2|\vec{a}| \cdot |\vec{b}| \cos\theta$

$= 1 + 1 - 2\cos\theta = 2(1 - \cos\theta) = 2 \cdot 2\sin^2\dfrac{\theta}{2}$

$\Rightarrow \quad \sin\dfrac{\theta}{2} = \dfrac{|\vec{a} - \vec{b}|}{2}$

56. (c) : We have, slope of curve $= \dfrac{2x}{y}$

$\Rightarrow \quad \dfrac{dy}{dx} = \dfrac{2x}{y} \Rightarrow \int y\,dy = \int 2x\,dx$

$\Rightarrow \quad y^2 = 2x^2 + C$ ...(i)

Now, (i) passes through (1, 2).

$\therefore \quad 4 = 2 + C \Rightarrow C = 2$

So, the required curve is, $y^2 - 2x^2 = 2$

$\Rightarrow \quad \dfrac{y^2}{2} - \dfrac{x^2}{1} = 1$, which represents hyperbola.

57. (c) : Let the position vectors of A, B and C are $0\hat{i} + 0\hat{j} + 0\hat{k}$, $\hat{i} + \hat{j} + \hat{k}$ and $\hat{i} + 3\hat{j} + 5\hat{k}$

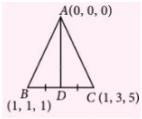
$\because$ AD is the median of ΔABC.

$\therefore$ D is the midpoint of BC.

$\Rightarrow$ Position vector of D

$= \left(\dfrac{(\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + 3\hat{j} + 5\hat{k})}{2}\right)$

$= \hat{i} + 2\hat{j} + 3\hat{k}$



$\therefore \quad \overrightarrow{AD} = (1-0)\hat{i} + (2-0)\hat{j} + (3-0)\hat{k} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\Rightarrow \quad |\overrightarrow{AD}| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$

58. (b)

59. (d) : Since the given vectors are coplanar.

$\therefore \quad \begin{vmatrix} 2 & -3 & 4 \\ 2 & 1 & -1 \\ \lambda & -1 & 2 \end{vmatrix} = 0$

$\Rightarrow \quad 2(2 - 1) + 3(4 + \lambda) + 4(-2 - \lambda) = 0$

$\Rightarrow \quad 2 + 12 + 3\lambda - 8 - 4\lambda = 0 \Rightarrow \lambda = 6$

60. (b) : We have,

$|\vec{a} \times \vec{b}|^2 + |\vec{a} \cdot \vec{b}|^2 = 144 \Rightarrow |\vec{a}|^2 \cdot |\vec{b}|^2 = 144$

$\Rightarrow \quad |\vec{b}|^2 = \dfrac{144}{36} = 4 \Rightarrow |\vec{b}| = 2$

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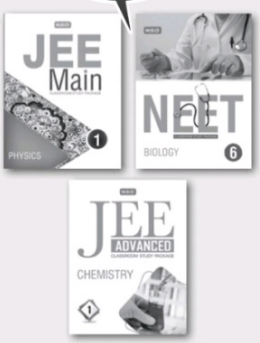

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CONCEPT BOOSTER

Class XI

Limits

This column is aimed for students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

*ALOK KUMAR

LIMITS

Consider the function $f(x)=\frac{x^2-1}{x-1}$. Clearly $f(x)$ is not defined at $x = 1$, since at $x = 1$, $f(x)=\frac{1-1}{1-1}=\frac{0}{0}$, which is meaningless.

x	0.9	0.99	0.999	0.9999	0.99999
$f(x)$	1.9	1.99	1.999	1.9999	1.99999

From the above it is clear that as x approaches to 1 *i.e.*, $x \to 1$ from the left hand side (means x approaches 1 from the values less than 1) $f(x)$ approaches to 2 *i.e.*, $f(x) \to 2$. The number 2 is called the left limit of $f(x)$ and in symbol we shall write $\lim_{x\to 1^-} f(x)=2$.

Again let us study the behaviour of $f(x)$ where x approaches towards 1 from the right-hand side.

x	1.1	1.01	1.001	1.0001	1.00001
$f(x)$	2.1	2.01	2.001	2.0001	2.00001

It is clear that as x approaches to 1 *i.e.* $x \to 1$, from the right-hand side (means x approaches 1 from the values greater than 1) $f(x)$ approaches to 2 *i.e.* $f(x) \to 2$. Here 2 is called the right-hand limit of $f(x)$ and in symbol we will write $\lim_{x\to 1^+} f(x)=2$.

Thus we see that $f(x)$ is not defined at $x = 1$ but its left-hand limit and right-hand limit as $x \to 1$ exist and are equal. When $\lim_{x\to 1^-} f(x)$ and $\lim_{x\to 1^+} f(x)$ are equal we say $\lim_{x\to 1} f(x)$ exist and is equal to 2.

- Meaning of $(x \to a)$: Let x be a variable and 'a' be a constant. x assumes value nearer and nearer to 'a', then we say 'x tends to a' and write '$x \to a$' and it doesn't mean $x = a$.
- Let $y = f(x)$ be a given function defined in the neighbourhood of $x = a$, but not necessarily at the point $x = a$. The limiting behaviour of the function in the neighbourhood of $x = a$ is called the limit of the function when x approaches to a. Mathematically, we write this as $\lim_{x\to a} f(x)$.

  $\lim_{x\to a} f(x)=l$ would simply mean that when we approach the point $x = a$ from the value which are just greater than or just smaller than $x = a$, $f(x)$ would have a tendency to move closer to the value l.

RIGHT AND LEFT HAND LIMIT

- Right-hand limit means tendency of function when we approach $x = a$ from the values which are just greater than 'a' and we write, it as $\lim_{x\to a^+} f(x)$.

  To evaluate $\lim_{x\to a^+} f(x)$, put $x = a + h$ in $f(x)$ to get $\lim_{h\to 0} f(a+h)$ and take the limit as $h \to 0$.
- Left-hand limit means tendency of function when we approach $x = a$ from the values which are just less than 'a' and we write it as $\lim_{x\to a^-} f(x)$.

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To evaluate $\lim_{x\to a^-} f(x)$, put $x = a - h$ in $f(x)$ to get $\lim_{h\to 0} f(a-h)$ and take the limit as $h \to 0$.

Example :

$$\lim_{x\to 1^+} \frac{x^2-1}{x-1} = \lim_{h\to 0} \frac{(1+h)^2-1}{(1+h)-1} = \lim_{h\to 0} \frac{h^2+2h}{h}$$
$$= \lim_{h\to 0} (h+2)$$

Thus for the existence of the limit of $f(x)$ at $x = a$, it is necessary and sufficient that $\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x)$, if these are finite or $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ both should be either $+\infty$ or $-\infty$.

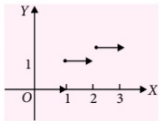
REASONS FOR NON-EXISTENCE OF THE LIMIT

Following are the reasons when $\lim_{x\to a} f(x)$ will not exist.

- When left and right tendencies of $f(x)$ are not same in the neighbourhood of $x = a$ *i.e.*, $\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$.

  For example, $\lim_{x\to 2} [x]$, (where $[\cdot]$ denotes the greatest integer function).

  It is clear from the figure that left tendency of $[x]$ at $x = 2$ is 1 while the right tendency of $[x]$ at $x = 2$ is 2. Thus $\lim_{x\to 2} [x]$, will not exist.



- If $f(x)$ is not defined in the neighbourhood of $x = a$. For example, $\lim_{x\to 0} \sec^{-1}(\cos x)$. Here $f(x)$ is well defined at $x = 0$ but in the left neighbourhood of 0, *i.e.*, $\lim_{x\to 0^-} \sec^{-1}(\cos x)$, when $x \to 0^-$, $\cos x < 1$ and $\sec^{-1}(\cos x)$ doesn't exist. Similarly, in the right neighbourhood of $x = 0$, *i.e.*, $\lim_{x\to 0^+} \sec^{-1}(\cos x)$, when $x \to 0^+$, $\cos x < 1$. Hence $\sec^{-1}(\cos x)$ is not defined.
- When $f(x)$ doesn't have a unique tendency. For example, $\lim_{x\to 0} \sin\frac{1}{x}$. We know that $-1 \le \sin x \le 1$, that means for all non-zero values of x, $\sin 1/x$ would assume finite values. But when x becomes very near to zero $\sin 1/x$ would erratically oscillate between -1 and $+1$. It implies that $\sin 1/x$ wouldn't have unique tendency for very small value of x. Thus $\lim_{x\to 0} \sin\frac{1}{x}$ will not exist.

BASIC THEOREMS ON LIMITS

Let $\lim_{x\to a} f(x) = l_1$ and $\lim_{x\to a} g(x) = l_2$, where l_1 and l_2 are finite, then the following theorems on limits can be used to evaluate the limits.

- $\lim_{x\to a} (c_1 f(x) \pm c_2 g(x)) = c_1 l_1 \pm c_2 l_2$, where c_1 and c_2 are given constants.
- $\lim_{x\to a} f(x)\cdot g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x) = l_1 l_2$
- $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} = \frac{l_1}{l_2}$, $l_2 \neq 0$
- If $f(x) \le g(x)\ \forall\ x \in R$, then $\lim_{x\to a} f(x) \le \lim_{x\to a} g(x)$.
- We have to be very careful while using these theorems. For example if we try to apply the theorems on $\lim_{x\to 0} \frac{\sin x}{x} = 1$, we get

  $\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \sin x \cdot \lim_{x\to 0} \frac{1}{x}$ which does not exist.

  Which is an absurd result, because in this case the given limit can not be written as the product of two limits as $\lim_{x\to 0} \frac{1}{x}$ does not exist.

EVALUATION OF LIMIT

- When the given limit is not in the indeterminate form, limit can be evaluated by directly using the definition of limit or putting a in place of x if $x \to a$. For example $\lim_{x\to 3} (x+3) = 6$. Here the form of limit is not indeterminate.

  When the form of the given function is indeterminate, then our motive will be to get rid of the indeterminate form. After getting rid of the indeterminate form we use the definition of limit or directly put the value.

  For example, $\lim_{x\to 2} \frac{x^2-4}{x-2}$ $\left(\frac{0}{0}\text{indeterminate form}\right)$

  $= \lim_{x\to 2} \frac{(x+2)(x-2)}{(x-2)} = \lim_{x\to 2} (x+2) = 4.$

METHOD TO EVALUATE THE LIMIT OF AN ALGEBRAIC FUNCTION

The following methods are useful for evaluating limits of algebraic function.

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- **Method of Factorisation:** If $f(x)$ and $g(x)$ are polynomials such that $f(a) = g(a) = 0$, then $(x - a)$ is a factor of both $f(x)$ and $g(x)$. Now to solve, $\lim_{x \to a} \frac{f(x)}{g(x)}$ we cancel the common factor $(x - a)$ from both the numerator and denominator, and again put $x = a$ in the given expression. If we get a meaningful number, then the number is the limit of the given expression otherwise we repeat the process till we get rid of indeterminate form.
- **Method of Rationalisation:** If in any limit denominator or numerator involve the radical sign, then this method is useful.

APPLICATION OF SOME BASIC LIMITS IN SOLVING THE LIMITS

Following are some basic limits which are used very frequently in solving the limits.

- If $p(x)$ is a polynomial, $\lim_{x \to a} p(x) = p(a)$
- $\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{\tan x}{x} = \lim_{x \to 0} \cos x = 1$ (where 'x' is in radians)
- $\lim_{x \to 0} (1+x)^{1/x} = e$
- $\lim_{x \to \infty} \left(1 + \frac{1}{x}\right)^x = e$
- $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$
- $\lim_{x \to 0} \frac{a^x - 1}{x} = \ln(a), a \in R^+$
- $\lim_{x \to 0} \frac{(1+x)^n - 1}{x} = n$
- $\lim_{x \to 0} \frac{x^m - a^m}{x^n - a^n} = \frac{m}{n} a^{m-n}$
- $\lim_{x \to 0} \frac{\ln(1+x)}{x} = 1$
- $\lim_{x \to 0} \frac{\log_a (1+x)}{x} = \log_a e, \ a > 0, \neq 1$
- $\lim_{x \to 0} \frac{\sin^{-1} x}{x} = 1 = \lim_{x \to 0} \frac{\tan^{-1} x}{x}$

Now if $\lim_{x \to a} f(x) = 0$ then we can redefine the limits in the following manner.

- $\lim_{x \to a} \frac{\sin f(x)}{f(x)} = \lim_{x \to a} \frac{\tan f(x)}{f(x)} = \lim_{x \to a} \cos f(x) = 1$
- $\lim_{x \to a} \frac{\sin^{-1} f(x)}{f(x)} = \lim_{x \to a} \frac{\tan^{-1} f(x)}{f(x)} = 1$
- $\lim_{x \to a} \frac{b^{f(x)} - 1}{f(x)} = \ln b \ (b > 0)$
- $\lim_{x \to a} (1 + f(x))^{1/f(x)} = e$

METHOD FOR CALCULATING THE LIMITS OF THE FORM $\lim_{x \to 0} f(x)^{g(x)}$

If the given limit is in the form $\lim_{x \to 0} f(x)^{g(x)}$, then we express the given expression as power of e. For this use the formula $a^x = e^{x \log_e a}$.

$$\lim_{x \to a} (f(x))^{g(x)} = e^{\lim_{x \to a} g(x) \log(f(x))} = e^{\lim_{x \to a} g(x) \lim_{x \to a} \log(f(x))}$$

$= e^{B(\log A)} = A^B$, where $\lim_{x \to a} g(x) = B$ and $\lim_{x \to a} f(x) = A$.

- If $f(x)$ is not throughout positive in the neighbourhood of $x = a$, then $\lim_{x \to a} (f(x))^{g(x)}$ will not exist because in this case function will not be defined in the neighbourhood of $x = a$.

METHOD TO SOLVE THE LIMIT OF THE FORM $\lim_{x \to \infty} a^x$

To solve the limit of the form $\lim_{x \to \infty} a^x$ we use the concept

$$\lim_{x \to \infty} a^x = \begin{cases} 0 & 0 \le a < 1 \\ 1 & a = 1 \\ \infty & a > 1 \end{cases}$$

- If $0 \le a < 1$, then a^x will keep on decreasing with increasing in the value of x. Hence $\lim_{x \to \infty} a^x = 0$
- If $a = 1$, in this case given function becomes a constant function. Hence $\lim_{x \to \infty} a^x = 1$.
- If $a > 1$, then a^x will keep on increasing with increase in the value of x. Hence $\lim_{x \to \infty} a^x = \infty$.
- If $a < 0$, then given function will not be defined. Hence $\lim_{x \to \infty} a^x = \infty$ will not exist.

L'HOSPITAL'S RULE

- Let $f(x)$ and $g(x)$ be two functions differentiable in the neighbourhood of the point a, except at the point a itself. If $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = 0$

  or, $\lim_{x \to a} f(x) = \lim_{x \to a} g(x) = \infty$,

  then $\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$ provided that the limit on the right exist as a finite number or is $\pm\infty$.
- L'Hospital's rule is not always useful. Consider the example, $\lim_{x \to \infty} \frac{x + \sin x}{x - \sin x} \left(\frac{\infty}{\infty} \text{ form}\right)$.

  Here, if we apply L'Hospital's rule, then

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$$\lim_{x\to\infty}\frac{x+\sin x}{x-\sin x}=\lim_{x\to\infty}\frac{1+\cos x}{1-\cos x}$$

Now, both the numerator and the denominator are undefined because $\lim_{x\to\infty}\cos x$ doesn't exist.

We can find this limit as follows:

$$\lim_{x\to\infty}\frac{x+\sin x}{x-\sin x}=\lim_{x\to\infty}\frac{1+\left(\frac{\sin x}{x}\right)}{1-\left(\frac{\sin x}{x}\right)}=\frac{1+0}{1-0}=1$$

$$\left(\because \lim_{x\to\infty}\frac{\sin x}{x}=0\right)$$

PROBLEMS

Single Correct Answer Type

1. $\lim_{x\to0}\frac{x(e^x-1)}{1-\cos x}=$

(a) 0 (b) ∞ (c) -2 (d) 2

2. $\lim_{x\to1}\frac{(2x-3)(\sqrt{x}-1)}{2x^2+x-3}=$

(a) $-1/10$ (b) $1/10$
(c) $-1/8$ (d) none of these

3. $\lim_{x\to0}\frac{1-\cos mx}{1-\cos nx}=$

(a) m/n (b) n/m (c) $\frac{m^2}{n^2}$ (d) $\frac{n^2}{m^2}$

4. $\lim_{x\to a}\frac{(x+2)^{5/3}-(a+2)^{5/3}}{x-a}=$

(a) $\frac{5}{3}(a+2)^{2/3}$ (b) $\frac{5}{3}(a+2)^{5/3}$
(c) $\frac{5}{3}a^{2/3}$ (d) $\frac{5}{3}a^{5/3}$

5. $\lim_{x\to0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}=$

(a) -1 (b) 1 (c) 2 (d) -2

6. $\lim_{x\to\infty}[x(a^{1/x}-1)],(a>1)=$

(a) $\log x$ (b) 1 (c) 0 (d) $-\log\frac{1}{a}$

7. $\lim_{x\to0}\frac{x\cos x-\sin x}{x^2\sin x}=$

(a) $\frac{1}{3}$ (b) $-\frac{1}{3}$
(c) 1 (d) none of these

8. $\lim_{x\to0}\left[\frac{1}{x}-\frac{\log(1+x)}{x^2}\right]=$

(a) 1/2 (b) $-1/2$ (c) 1 (d) -1

9. $\lim_{\theta\to0}\frac{5\theta\cos\theta-2\sin\theta}{3\theta+\tan\theta}=$

(a) $\frac{3}{4}$ (b) $-\frac{3}{4}$
(c) 0 (d) none of these

10. $\lim_{x\to0}\frac{\log(a+x)-\log a}{x}+k\lim_{x\to e}\frac{\log x-1}{x-e}=1$, then

(a) $k=e\left(1-\frac{1}{a}\right)$ (b) $k=e(1+a)$
(c) $k=e(2-a)$
(d) The equality is not possible

11. If a, b, c, d are positive, then

$$\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}=$$

(a) $e^{d/b}$ (b) $e^{c/a}$
(c) $e^{(c+d)/(a+b)}$ (d) e

12. $\lim_{n\to\infty}(4^n+5^n)^{1/n}$ is equal to

(a) 4 (b) 5
(c) $\alpha^2-\beta^2$ (d) none of these

13. $\lim_{x\to0}\frac{e^{\alpha x}-e^{\beta x}}{x}=$

(a) $\alpha+\beta$ (b) $\frac{1}{\alpha}+\beta$ (c) $\alpha^2-\beta^2$ (d) $\alpha-\beta$

14. $\lim_{x\to0}\left(\frac{1+\tan x}{1+\sin x}\right)^{\operatorname{cosec} x}$ is equal to

(a) e (b) $1/e$
(c) 1 (d) none of these

15. $\lim_{x\to a}f(x)\cdot g(x)$ exists, if

(a) $\lim_{x\to a}f(x)$ and $\lim_{x\to a}g(x)$ exist

(b) $\lim_{x\to a}f(x)^{g(x)}$ exist (c) $\lim_{x\to a}\frac{f(x)}{g(x)}$ exist

(d) $\lim_{x\to a}f(x)g\left(\frac{1}{x}\right)$ exist

16. $\lim_{x\to0}\frac{e^{\tan x}-e^x}{\tan x-x}=$

(a) 1 (b) e (c) e^{-1} (d) 0

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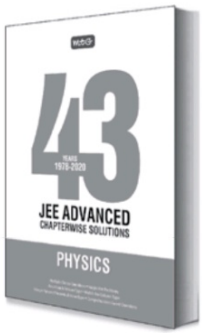

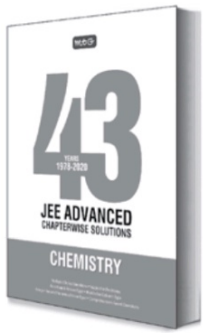

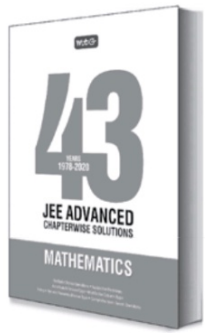

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17. $\lim_{x\to 0}\left[\frac{\sin(x+a)+\sin(a-x)-2\sin a}{x\sin x}\right]=$

(a) $\sin a$ (b) $\cos a$

(c) $-\sin a$ (d) $\frac{1}{2}\cos a$

18. The value of $\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{2/x}$; $(a, b, c > 0)$ is

(a) $(abc)^3$ (b) abc

(c) $(abc)^{1/3}$ (d) none of these

19. $\lim_{x\to \pi/2}\left[x\tan x-\left(\frac{\pi}{2}\right)\sec x\right]=$

(a) 1 (b) -1

(c) 0 (d) none of these

20. If $f(x)=\begin{cases}x, & \text{if } x \text{ is rational}\\ -x, & \text{if } x \text{ is irrational}\end{cases}$, then $\lim_{x\to 0} f(x)$ is

(a) equal to 0 (b) equal to 1

(c) equal to -1 (d) indeterminate

21. If $f(x)=\begin{cases}\sin x, x\neq n\pi, n\in Z\\ 0, \text{ otherwise}\end{cases}$ and

$g(x)=\begin{cases}x^2+1, x\neq 0,2\\ 4, x=0\\ 5, x=2\end{cases}$, then $\lim_{x\to 0} g\{f(x)\}=$

(a) 1 (b) 0 (c) 1/2 (d) 1/4

22. $\lim_{x\to 0}\frac{x\tan 2x-2x\tan x}{(1-\cos 2x)^2}$ is

(a) 2 (b) -2 (c) 1/2 (d) $-1/2$

23. The value of $\lim_{x\to 0}\frac{(1-\cos 2x)\sin 5x}{x^2\sin 3x}$ is

(a) 10/3 (b) 3/10 (c) 6/5 (d) 5/6

24. The value of $\lim_{x\to 0}\frac{x\cos x-\log(1+x)}{x^2}$ is

(a) 1/2 (b) 0

(c) 1 (d) none of these

25. $\lim_{x\to 0}\frac{\ln(\cos x)}{x^2}$ is equal to

(a) 0 (b) 1 (c) 1/2 (d) $-1/2$

26. The value of $\lim_{a\to 0}\frac{\sin a-\tan a}{\sin^3 a}$ will be

(a) $-1/2$ (b) 1/2 (c) 1 (d) -1

27. For $x\in R$, $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to

(a) e (b) e^{-1}

(c) e^{-5} (d) e^5

28. The value of $\lim_{x\to 0}\left[\frac{\sqrt{a+x}-\sqrt{a-x}}{x}\right]$ is

(a) 1 (b) 0 (c) $\sqrt{a}$ (d) $1/\sqrt{a}$

29. $\lim_{x\to\infty}\frac{\log x^n-[x]}{[x]}, n\in N$, ($[x]$ denotes greatest integer less than or equal to x)

(a) has value -1 (b) has value 0

(c) has value 1 (d) does not exist

30. $\lim_{n\to\infty}\left(\frac{n^2-n+1}{n^2-n-1}\right)^{n(n-1)}=$

(a) e (b) e^2 (c) e^{-1} (d) 1

31. The value of the constant α and β such that $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-\alpha x-\beta\right)=0$ are respectively

(a) (1, 1) (b) (−1, 1) (c) (1, −1) (d) (0, 1)

32. If $a_1 = 1$ and $a_{n+1}=\frac{4+3a_n}{3+2a_n}, n\geq 1$ and if $\lim_{n\to\infty} a_n = a$, then the value of a is

(a) $\sqrt{2}$ (b) $-\sqrt{2}$

(c) 2 (d) none of these

33. The value of

$\lim_{n\to\infty}\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{8}\right)\ldots\cos\left(\frac{x}{2^n}\right)$ is

(a) 1 (b) $\frac{\sin x}{x}$

(c) $\frac{x}{\sin x}$ (d) none of these

34. $\lim_{n\to\infty}\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\ldots+\frac{1}{2^n}$ equals

(a) 2 (b) -1 (c) 1 (d) 3

Monthly Test Drive-6 CLASS XI ANSWER KEY

1. (b) 2. (a) 3. (d) 4. (b) 5. (a)
6. (b) 7. (a,b,c,d) 8. (a,b) 9. (a,b,c)
10. (a,b,d) 11. (a,c) 12. (a,b,c) 13. (a,d)
14. (b) 15. (c) 16. (c) 17. (7) 18. (5)
19. (8) 20. (1)

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35. Let $f : R \to R$ be a differentiable function having $f(2) = 6, f'(2) = \left(\frac{1}{48}\right)$, then $\lim_{x\to 2} \frac{\int_6^{f(x)} 4t^3\,dt}{x-2}$ is equal to

(a) 12 (b) 18 (c) 24 (d) 36

36. $\lim_{x\to\infty} \frac{(x+1)^{10}+(x+2)^{10}+\ldots+(x+100)^{10}}{x^{10}+10^{10}}$ is equal to

(a) 0 (b) 1 (c) 10 (d) 100

37. The value of $\lim_{x\to 0} \frac{\int_0^x \cos t^2\,dt}{x}$ is

(a) 0 (b) 1
(c) −1 (d) none of these

38. $\lim_{n\to\infty}\left[\frac{1}{n^3+1}+\frac{4}{n^3+1}+\frac{9}{n^3+1}+\ldots\ldots+\frac{n^2}{n^3+1}\right] =$

(a) 1 (b) 2/3 (c) 1/3 (d) 0

39. If $x_n = \frac{1-2+3-4+5-6+\ldots..-2n}{\sqrt{n^2+1}+\sqrt{4n^2-1}}$, then $\lim_{n\to\infty} x_n$ is equal to

(a) 1/3 (b) −2/3 (c) 2/3 (d) 1

Assertion & Reason Type

Directions : *In the following questions, Statement-1 is followed by Statement-2. Mark the correct choice as :*

(a) Statement-1 is true, Statement-2 is true, Statement-2 is a correct explanation for Statement-1.

(b) Statement-1 is true, Statement–2 is true, Statement-2 is not a correct explanation for Statement-1

(c) Statement-1 is true, Statement-2 is false

(d) Statement-1 is false, Statement-2 is true

40. Statement-1:

$$\lim_{x\to\infty} \frac{(x+1)^{10}+(x+2)^{10}+\ldots..+(x+100)^{10}}{x^{10}+9^{10}} = 100$$

Statement-2: If $p(x)$ and $q(x)$ are polynomials of same degree, then

$$\lim_{x\to\infty} \frac{p(x)}{q(x)} = \frac{\text{leading coefficient of } p(x)}{\text{leading coefficient of } q(x)}$$

41. Statement-1: $\lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x$ is equal to e^{-5}.

Statement-2: $\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x = e$

42. Statement-1: Let $f(x) = \begin{cases} \frac{\sin x}{x}, & \text{if } x<0 \\ ax+b, & \text{if } x>0 \end{cases}$

If $\lim_{x\to 0} f(x)$ exists, then $a = 1$ and $b = 0$.

Statement-2: $\lim_{x\to 0}(px+q) = q$, where p and q are any real constants.

43. Statement-1: If a and b are positive and $[x]$ denotes the greatest integer $\le x$, then $\lim_{x\to 0^+} \frac{x}{a}\left[\frac{b}{x}\right] = \frac{b}{a}$.

Statement-2: $\lim_{x\to\infty} \frac{\{x\}}{x} = 0$, where $\{x\}$ denotes fractional part of x.

44. Statement-1: If $\lim_{x\to 0}\left(\frac{\sin 2x + a\sin x}{x^3}\right)$ exists finitely, then the value of a is −2.

Statement-2: If $\lim_{x\to a} \frac{f(x)}{g(x)}$ exists finitely and $\lim_{x\to a} g(x) = 0$ then $\lim_{x\to a} f(x) = 0$.

Comprehension Type

Paragraph for Q. No. 45 to 47

In the evaluation of limits in the following passage, one may use one or the other of the following results:

(i) If $f(x)$ and $g(x)$ are functions defined in some deleted neighbourhood N of 'a' such that $g(x)$ never vanishes in N and $\lim_{x\to a} \frac{f(x)}{g(x)}$ exists, then $\lim_{x\to a} g(x) = 0$ implies $\lim_{x\to a} f(x) = 0$

(ii) If $f(x) \to 1$ as $x \to a$, $g(x) \to +\infty$ as $x \to a$ and $\lim_{x\to a}\{f(x)\}^{g(x)}$ exists, then this limit equals $e^{\lim_{x\to a} g(x)\{f(x)-1\}}$.

45. If $\lim_{x\to 0} \frac{\sin ax - \log_e(e^x \cos x)}{x\sin bx} = \frac{1}{2}$, then

(a) $a = 1, b = 1$ (b) $a = -1, b = -1$
(c) $a = -1, b = 1$ (d) $a = -1, b = 1/2$

46. Let $f(x)$ be a function that is defined in a deleted neighbourhood of '0' such that $\lim_{x\to 0} \frac{f(x)}{x} = 1$. If $\lim_{x\to 0} \frac{x(1+a\cos x) - b\sin x}{\{f(x)\}^3} = 1$, then

(a) $a = -\frac{3}{2}, b = -\frac{1}{2}$ (b) $a = \frac{1}{2}, b = \frac{3}{2}$

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(c) $a=-\frac{5}{2}, b=-\frac{3}{2}$ (d) $a=\frac{3}{2}, b=\frac{5}{2}$

47. Let $L=\lim_{x\to 0}\dfrac{a-\sqrt{a^2-x^2}-\frac{x^2}{2}}{x^2\sin^2 x}, a>0$. If L is finite, then $L=$

(a) 1/4 (b) −1/4 (c) 1/8 (d) −1/8

Paragraph for Q. No. 48 to 50

Let $f(x)={}^nC_x p^x q^{n-x}; x=0,1,2,.....,n$ where $p+q=1$, $0<p<1$.

Here ${}^nC_x=\dfrac{n!}{x!(n-x)!}$ and $n!=1\times 2\times 3\times ...\times n$, $np=\lambda$ (finite) when $p\to 0$ and $n\to\infty$.

48. $\lim_{n\to\infty}\sum_{x=0}^{n} x f(x)=$

(a) λ (b) $\lambda/2$ (c) 0 (d) 1

49. $\lim_{n\to\infty}\sum_{x=0}^{n} x^2 f(x)=$

(a) $1+\lambda$ (b) $\lambda+\lambda^2$ (c) λ^2 (d) 0

50. $\lim_{n\to\infty}\sum_{x=0}^{n} (x-\lambda)^2 f(x)=$

(a) λ (b) 0 (c) 2λ (d) $\lambda^2+\lambda$

Matrix-Match Type

51. Match the following :

	Column-I		Column-II
A.	If $f(x)=x\ \text{sgn}\ (x-1)$, then	(p)	$\lim_{x\to 1} f(x)$ does not exist
B.	If $f(x)=\dfrac{\sin\left(\sin(\tan(x^2/2))\right)}{\log\cos 3x}$, then	(q)	$\lim_{x\to 0} f(x)$ does not exist
C.	If $f(x)=\dfrac{\sqrt[3]{1+\tan^{-1}3x}-\sqrt[3]{1-\sin^{-1}3x}}{\sqrt{1-\sin^{-1}2x}-\sqrt{1+\tan^{-1}2x}}$, then	(r)	$\lim_{x\to 0} f(x)=\dfrac{-1}{9}$
D.	If $f(x)=\dfrac{e^{1/x}-1}{e^{1/x}+1}$, then	(s)	$\lim_{x\to 0} f(x)=-1$

52. Match the following :

	Column-I		Column-II
A.	$\lim_{x\to 3}\dfrac{(x^3+27)\log(x-2)}{x^2-9}=$	(p)	12
B.	$\lim_{x\to 0}\left(\dfrac{e^x-1}{x}\right)^{\left(\frac{x}{x+1-e^x}\right)}=$	(q)	8
C.	If $\lim_{x\to 0}\dfrac{x(a+\cos x)-b\sin x}{x^3}=1$, then a and b are respectively	(r)	9
D.	If $f(x)$ is a thrice differentiable function such that $\lim_{x\to 0}\dfrac{f(4x)-3f(3x)+3f(2x)-f(x)}{x^3}=12$ then $f'''(0)$ is equal to	(s)	e^{-1}

SOLUTIONS

1. (d): $\lim_{x\to 0}\dfrac{x(e^x-1)}{1-\cos x}=\lim_{x\to 0}\dfrac{2x(e^x-1)}{4\cdot\sin^2\frac{x}{2}}$

$=2\lim_{x\to 0}\left[\dfrac{(x/2)^2}{\sin^2\frac{x}{2}}\right]\left(\dfrac{e^x-1}{x}\right)=2$

2. (a): $\lim_{x\to 1}\dfrac{(2x-3)(\sqrt{x}-1)\times(\sqrt{x}+1)}{(x-1)(2x+3)\times(\sqrt{x}+1)}=\dfrac{-1}{5\times 2}=\dfrac{-1}{10}$

3. (c): $\lim_{x\to 0}\dfrac{1-\cos mx}{1-\cos nx}=\lim_{x\to 0}\left\{\dfrac{2\sin^2\frac{mx}{2}}{2\sin^2\frac{nx}{2}}\right\}$

$=\lim_{x\to 0}\left[\left\{\dfrac{\sin\frac{mx}{2}}{\frac{mx}{2}}\right\}^2\dfrac{m^2x^2}{4}\cdot\dfrac{1}{\left\{\frac{\sin\frac{nx}{2}}{\frac{nx}{2}}\right\}^2}\cdot\dfrac{4}{n^2x^2}\right]$

$=\dfrac{m^2}{n^2}\times 1=\dfrac{m^2}{n^2}$

4. (a): $\lim_{x\to a}\dfrac{(x+2)^{5/3}-(a+2)^{5/3}}{(x-a)}$

$=\lim_{x\to a}\dfrac{\frac{5}{3}(x+2)^{2/3}-0}{1-0}$ [By L'Hospital's rule] $=\dfrac{5}{3}(a+2)^{2/3}$

5. (b): $\lim_{x\to 0}\dfrac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{x}$

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Applying L'Hospital's rule, we have

$$\lim_{x\to 0}\frac{\frac{1}{2\sqrt{1+\sin x}}\cdot\cos x+\frac{\cos x}{2\sqrt{1-\sin x}}}{1}$$

$$=\frac{1}{2}\lim_{x\to 0}\left(\sqrt{1-\sin x}+\sqrt{1+\sin x}\right)\ =\frac{1}{2}\cdot 2=1$$

6. (d): $\lim_{x\to\infty} x[a^{1/x}-1]=\lim_{x\to\infty}\left[\frac{a^{1/x}-1}{1/x}\right]$

$$=\lim_{x\to\infty}\frac{[e^{\log_e a^{1/x}}-1]}{1/x}=\log_e a=-\log_e\frac{1}{a}$$

7. (b): $\lim_{x\to 0}\frac{x\cos x-\sin x}{x^2\sin x}$

$$=\lim_{x\to 0}\frac{-\sin x}{2\sin x+x\cos x}$$ (By L'Hospital's rule)

$$=\lim_{x\to 0}\frac{-\cos x}{3\cos x-x\sin x}$$ (Again by L'Hospital's rule)

$$=-\frac{1}{3}$$

8. (a): $\lim_{x\to 0}\left[\frac{1}{x}-\frac{\log(1+x)}{x^2}\right]=\lim_{x\to 0}\left[\frac{x-\log(1+x)}{x^2}\right]$

$$=\lim_{x\to 0}\frac{1-\frac{1}{1+x}}{2x}$$ [Applying L'Hospital's rule]

$$=\lim_{x\to 0}\frac{1}{2}\left(\frac{1}{1+x}\right)=\frac{1}{2}$$

9. (a): $\lim_{\theta\to 0}\frac{5\theta\cos\theta-2\sin\theta}{3\theta+\tan\theta}$

$$=\lim_{\theta\to 0}\frac{5\cos\theta-\frac{2\sin\theta}{\theta}}{3+\frac{\tan\theta}{\theta}}=\frac{5-2}{3+1}=\frac{3}{4}$$

10. (a): Let $f(x)=\log x \Rightarrow f'(x)=\frac{1}{x}$

Therefore, given function $= f'(a) + kf'(e) = 1$

$$\Rightarrow\ \frac{1}{a}+\frac{k}{e}=1\ \Rightarrow\ k=e\left(\frac{a-1}{a}\right)$$

11. (a) $\lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{c+dx}$

$$=\lim_{x\to\infty}\left\{\left(1+\frac{1}{a+bx}\right)^{a+bx}\right\}^{\frac{c+dx}{a+bx}}=e^{d/b}$$

$$\left\{\because\ \lim_{x\to\infty}\left(1+\frac{1}{a+bx}\right)^{a+bx}=e \text{ and } \lim_{x\to\infty}\frac{c+dx}{a+bx}=\frac{d}{b}\right\}$$

12. (b): Given limit $= \lim_{n\to\infty}(4^n+5^n)^{1/n}$

$$=\lim_{n\to\infty}5\left[\left\{1+\left(\frac{4}{5}\right)^n\right\}^{(5/4)^n}\right]^{(1/n)\cdot(4/5)^n}=5\cdot e^0=5$$

$$\left(\because\ \left(\frac{4}{5}\right)^n\to 0 \text{ as } n\to\infty\right)$$

13. (d): $\lim_{x\to 0}\frac{e^{\alpha x}-e^{\beta x}}{x}=\lim_{x\to 0}\frac{e^{\alpha x}-1-e^{\beta x}+1}{x}$

$$=\alpha\lim_{x\to 0}\frac{e^{\alpha x}-1}{\alpha x}-\beta\lim_{x\to 0}\frac{e^{\beta x}-1}{\beta x}=\alpha\cdot 1-\beta\cdot 1=\alpha-\beta$$

14. (c): Given limit

$$=\lim_{x\to 0}[(1+\tan x)^{\operatorname{cosec} x}\times 1/(1+\sin x)^{\operatorname{cosec} x}]$$

$$=\lim_{x\to 0}[\{(1+\tan x)^{\cot x}\}^{\sec x}\times\{1/(1+\sin x)^{\operatorname{cosec} x}\}]$$

$$=e^{\sec 0}.\frac{1}{e}=e.\frac{1}{e}=1$$

15. (a): The limit of product exists if the product of limits exist.

16. (a): $\lim_{x\to 0}\frac{e^{\tan x}-e^x}{\tan x-x}=\lim_{x\to 0}\frac{e^x[e^{\tan x-x}-1]}{\tan x-x}$

$$=\lim_{x\to 0}e^x.\lim_{x\to 0}\frac{e^{\tan x-x}-1}{\tan x-x}=e^0\times 1=1$$

17. (c): Given limit $= \lim_{x\to 0}2\sin a.\frac{(\cos x-1)}{x\sin x}$

$$=\lim_{x\to 0}(-2\sin a).\frac{(1-\cos x)}{x^2}.\left(\frac{x}{\sin x}\right)$$

$$=\lim_{x\to 0}(-2\sin a).\frac{2\sin^2(x/2)}{4\left(\frac{x}{2}\right)^2\left(\frac{\sin x}{x}\right)}=-\sin a$$

18. (d): Let $y=\lim_{x\to 0}\left(\frac{a^x+b^x+c^x}{3}\right)^{2/x}$

$$\therefore\ \log y=\lim_{x\to 0}\log\left(\frac{a^x+b^x+c^x}{3}\right)^{2/x}$$

$$=2\lim_{x\to 0}\frac{\log(a^x+b^x+c^x)-\log 3}{x}$$

Now applying L'Hospital's rule, we have

$\log y = \log(abc)^{2/3} \Rightarrow y=(abc)^{2/3}$

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19. (b) : Given, $\lim_{x\to\pi/2}\left[x\tan x-\left(\frac{\pi}{2}\right)\sec x\right]$

$$=\lim_{x\to\pi/2}\frac{2x\sin x-\pi}{2\cos x}\quad\left(\frac{0}{0}\text{ form}\right)$$

Applying L'Hospital's rule, we have

$$\lim_{x\to\pi/2}\left(\frac{2\sin x+2x\cos x}{-2\sin x}\right)=-1$$

20. (a) : $\lim_{x\to0^-}f(x)=\lim_{h\to0}f(0-h)=\lim_{h\to0}(0-h)=0$

and $\lim_{x\to0^+}f(x)=\lim_{h\to0}f(0+h)=\lim_{h\to0}-(0+h)=0$

$\therefore\ \lim_{x\to0}f(x)=0\ \left(\because\lim_{x\to0^-}f(x)=\lim_{x\to0^+}f(x)\right)$

21. (a) : We have, $\lim_{x\to0}g\{f(x)\}$

$$=\lim_{x\to0}\left[\{f(x)\}^2+1\right]=\lim_{x\to0}(\sin^2x+1)=1$$

22. (c) : $\lim_{x\to0}\frac{x\tan2x-2x\tan x}{(1-\cos2x)^2}$

$$=\lim_{x\to0}\frac{x(\tan2x-2\tan x)}{(2\sin^2x)^2}=\lim_{x\to0}\frac{1}{4}\frac{x(\tan2x-2\tan x)}{\sin^4x}$$

$$=\lim_{x\to0}\frac{1}{4}\frac{x\left\{\left(2x+\frac{1}{3}(2x)^3+\frac{2}{15}(2x)^5+\ldots\right)-2\left(x+\frac{x^3}{3}+\frac{2}{15}x^5+\ldots\right)\right\}}{x^4\left(1-\frac{x^2}{3!}+\frac{x^4}{5!}-\ldots.\right)^4}$$

$$=\frac{1}{4}\cdot\left(\frac{8}{3}-\frac{2}{3}\right)=\frac{2}{4}=\frac{1}{2}$$

23. (a) : $\lim_{x\to0}\frac{(1-\cos2x)\sin5x}{x^2\sin3x}=\lim_{x\to0}\frac{2\sin^2x\sin5x}{x^2\sin3x}$

$$=\lim_{x\to0}\left(\frac{2\sin^2x}{x^2}\right)\frac{\left(\frac{\sin5x}{x}\right)}{\left(\frac{\sin3x}{x}\right)}$$

$$=\lim_{x\to0}2\left(\frac{\sin x}{x}\right)^2\times\frac{5\lim_{x\to0}\left(\frac{\sin5x}{5x}\right)}{3\lim_{x\to0}\left(\frac{\sin3x}{3x}\right)}=\frac{2\times5}{3}=\frac{10}{3}$$

24. (a) : Given, $\lim_{x\to0}\frac{x\cos x-\log(1+x)}{x^2}\quad\left(\frac{0}{0}\text{ form}\right)$

Applying L'Hospital's rule, we have

$$\lim_{x\to0}\frac{\cos x-x\sin x-\frac{1}{x+1}}{2x}\quad\left(\frac{0}{0}\text{ form}\right)$$

$$=\lim_{x\to0}\frac{-\sin x-\sin x-x\cos x+\frac{1}{(x+1)^2}}{2}=\frac{1}{2}$$

25. (d) : Given, $\lim_{x\to0}\frac{\ln(\cos x)}{x^2}\quad\left(\frac{0}{0}\text{ form}\right)$

Applying L'Hospital's rule, we have

$$\lim_{x\to0}\frac{-\tan x}{2x}\left(\frac{0}{0}\text{ form}\right)=\lim_{x\to0}\frac{-\sec^2x}{2}=\frac{-1}{2}$$

26. (a) : We have, $\lim_{a\to0}\frac{\sin a-\tan a}{\sin^3a}$

$$=\lim_{a\to0}\frac{\cos a-1}{\sin^2a\cos a}=\lim_{a\to0}\frac{-(1-\cos a)}{(1-\cos^2a)(\cos a)}$$

$$=\lim_{a\to0}\left[-\frac{1}{(1+\cos a)\cos a}\right]=-\frac{1}{(1+1)1}=\frac{-1}{2}$$

27. (c) : $\lim_{x\to\infty}\left(\frac{x+2-5}{x+2}\right)^x$

$$=\lim_{x\to\infty}\left[\left(1-\frac{5}{x+2}\right)^{\frac{x+2}{-5}}\right]^{\frac{5x}{x+2}}=e^{-5}$$

$$\left(\because\lim_{x\to\infty}\frac{-5x}{x+2}=\lim_{x\to\infty}\frac{-5}{1+\frac{2}{x}}=-5\right)$$

28. (d) : $\lim_{x\to0}\left[\frac{\sqrt{a+x}-\sqrt{a-x}}{x}\right]$

$$=\lim_{x\to0}\left[\frac{(\sqrt{a+x}-\sqrt{a-x})(\sqrt{a+x}+\sqrt{a-x})}{x(\sqrt{a+x}+\sqrt{a-x})}\right]$$

$$=\lim_{x\to0}\left[\frac{2x}{x(\sqrt{a+x}+\sqrt{a-x})}\right]=\frac{2}{\sqrt{a}+\sqrt{a}}=\frac{1}{\sqrt{a}}$$

29. (a) : We have, $\lim_{x\to\infty}\frac{\log x^n-[x]}{[x]}$

$$=\lim_{x\to\infty}\frac{\log x^n}{[x]}-\lim_{x\to\infty}\frac{[x]}{[x]}\ =0-1=-1$$

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30. (b) : We have, $\lim_{n\to\infty}\left(\frac{n^2-n+1}{n^2-n-1}\right)^{n(n-1)}$

$$= \lim_{n\to\infty}\left(\frac{n(n-1)+1}{n(n-1)-1}\right)^{n(n-1)}$$

$$= \lim_{n\to\infty}\frac{\left(1+\frac{1}{n(n-1)}\right)^{n(n-1)}}{\left(1-\frac{1}{n(n-1)}\right)^{n(n-1)}} = \frac{e}{e^{-1}} = e^2$$

31. (c) : Given, $\lim_{x\to\infty}\left(\frac{x^2+1}{x+1}-\alpha x-\beta\right)=0$

$$\Rightarrow \lim_{x\to\infty}\frac{x^2(1-\alpha)-x(\alpha+\beta)+1-\beta}{x+1}=0$$

Since the limit of the given expression is zero, therefore degree of the polynomial in numerator must be less than denominator.

$\therefore$ $1-\alpha=0$ and $\alpha+\beta=0 \Rightarrow \alpha=1$ and $\beta=-1$

32. (a) : We have $a_{n+1}=\frac{4+3a_n}{3+2a_n}$

$$\Rightarrow \lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty}\frac{4+3a_n}{3+2a_n}$$

$$\Rightarrow a=\frac{4+3a}{3+2a} \Rightarrow 2a^2=4 \Rightarrow a=\sqrt{2}$$

[$\because$ $a\neq-\sqrt{2}$ because each $a_n>0$]

Therefore, $\lim_{n\to\infty} a_n = a > 0$

33. (b) : We know that

$$\cos A\cos 2A\cos 4A....\cos 2^{n-1}A = \frac{\sin 2^n A}{2^n \sin A}$$

Taking $A=\frac{x}{2^n}$, we get

$$\cos\left(\frac{x}{2^n}\right)\cos\left(\frac{x}{2^{n-1}}\right)....\cos\left(\frac{x}{4}\right)\cos\left(\frac{x}{2}\right)=\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)}$$

$$\therefore \lim_{n\to\infty}\cos\left(\frac{x}{2}\right)\cos\left(\frac{x}{4}\right)...\cos\left(\frac{x}{2^{n-1}}\right)\cos\left(\frac{x}{2^n}\right)$$

$$= \lim_{n\to\infty}\frac{\sin x}{2^n\sin\left(\frac{x}{2^n}\right)} = \lim_{n\to\infty}\frac{\sin x}{x}\frac{(x/2^n)}{\sin(x/2^n)} = \frac{\sin x}{x}$$

34. (c) : Let $y=\lim_{n\to\infty}\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+.....+\frac{1}{2^n}$

$$= \lim_{n\to\infty}\frac{1}{2}\frac{\left[1-\left(\frac{1}{2}\right)^n\right]}{\left(1-\frac{1}{2}\right)} = \lim_{n\to\infty}\left[1-\frac{1}{2^n}\right]=1-0=1$$

35. (b) : $\lim_{x\to 2}\frac{\int_6^{f(x)} 4t^3\,dt}{x-2}$ $\left(\frac{0}{0}\text{ form}\right)$

$$= \lim_{x\to 2}\frac{4(f(x))^3\times f'(x)}{1} = 4(f(2))^3\times f'(2) = 18$$

35. (d) : $\lim_{x\to\infty}\frac{(x+1)^{10}+(x+2)^{10}+.....+(x+100)^{10}}{x^{10}+10^{10}}$

$$= \lim_{x\to\infty}\frac{x^{10}\left[\left(1+\frac{1}{x}\right)^{10}+\left(1+\frac{2}{x}\right)^{10}+...+\left(1+\frac{100}{x}\right)^{10}\right]}{x^{10}\left[1+\frac{10^{10}}{x^{10}}\right]}=100$$

37. (b) : We have, $\lim_{x\to 0}\frac{\int_0^x \cos t^2\,dt}{x}$

Applying L'Hospital's rule, we have $\lim_{x\to 0}\frac{\cos x^2}{1}=1$

38. (c) : Given limit $= \lim_{n\to\infty}\frac{1^2+2^2+3^2+.....+n^2}{1+n^3}$

$$= \lim_{n\to\infty}\frac{\Sigma n^2}{1+n^3} = \lim_{n\to\infty}\frac{1}{6}\cdot\frac{n(n+1)(2n+1)}{1+n^3}$$

$$= \lim_{n\to\infty}\frac{1}{6}\frac{\left(1+\frac{1}{n}\right)\left(2+\frac{1}{n}\right)}{\left(\frac{1}{n^3}+1\right)} = \frac{1}{6}\cdot\frac{1.2}{1}=\frac{1}{3}$$

39. (b) : $\lim_{n\to\infty}\frac{1-2+3-4+5-6+.....-2n}{\sqrt{n^2+1}+\sqrt{4n^2-1}}$

$$= \lim_{n\to\infty}\frac{n\left[\left(\frac{1}{n}-\frac{2}{n}+\frac{3}{n}-....\right)-2\right]}{n\left[\sqrt{1+\frac{1}{n^2}}+\sqrt{4-\frac{1}{n^2}}\right]} = \frac{-2}{1+2}=\frac{-2}{3}$$

40. (a) : Let $p(x) = a_0x^n + a_1x^{n-1} + ... + a_n$ and $q(x) = b_0x^n + b_1x^{n-1} + ... + b_n$, then

$$\lim_{x\to\infty}\frac{p(x)}{q(x)} = \lim_{x\to\infty}\frac{a_0+a_1/x+a_2/x^2+...+a_n/x^n}{b_0+b_1/x+b_2/x^2+...+b_n/x^n}$$

$$= \frac{a_0}{b_0}$$

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$(x+1)^{10}+(x+2)^{10}+\ldots+(x+100)^{10}$ is a polynomial of degree 10 with leading coefficient 100 and $x^{10}+9^{10}$ is a polynomial of degree 10 with leading coefficient 1. So Statement-1 follows from Statement-2.

41. (a) : Statement-2 is clearly true.

$$\text{Now, } \lim_{x\to\infty}\left(\frac{x-3}{x+2}\right)^x=\lim_{x\to\infty}\left(\frac{1-\frac{3}{x}}{1+\frac{2}{x}}\right)^x$$

$$=\frac{\lim\limits_{x\to\infty}\left[\left(1-\frac{3}{x}\right)^{-x/3}\right]^{-3}}{\lim\limits_{x\to\infty}\left[\left(1+\frac{2}{x}\right)^{x/2}\right]^2}=\frac{e^{-3}}{e^2}=e^{-5}$$

Statement-1 is also true and Statement-2 is a correct explanation of Statement-1.

42. (d) : Statement-2 is clearly true.

Now, $\lim\limits_{x\to0} f(x)$ exists, so the left and right limits of $f(x)$ at $x=0$ must be equal.

$$\text{Now, } \lim_{x\to0^-} f(x)=\lim_{x\to0^-}\left(\frac{\sin x}{x}\right)=1$$

$$\text{Also, } \lim_{x\to0^+} f(x)=\lim_{x\to0^+}(ax+b)=b$$

Therefore $b=1$ whereas a may be any real number. Hence Statement-1 is false.

43. (a) : We have, $\lim\limits_{x\to0^+}\frac{x}{a}\left[\frac{b}{x}\right]=\lim\limits_{x\to0^+}\frac{x}{a}\left(\frac{b}{x}-\left\{\frac{b}{x}\right\}\right)$

$$=\lim_{x\to0^+}\frac{x}{a}\cdot\frac{b}{x}\left(1-\frac{\left\{\frac{b}{x}\right\}}{\left(\frac{b}{x}\right)}\right)=\lim_{x\to0^+}\frac{b}{a}\left(1-\frac{\left\{\frac{b}{x}\right\}}{\left(\frac{b}{x}\right)}\right)$$

$$=\frac{b}{a}-\frac{b}{a}\lim_{y\to\infty}\frac{\{y\}}{y},\quad\left(\text{where } y=\frac{b}{x}\right)=\frac{b}{a}$$

Since, $0\le\{x\}<1$ so $\frac{\{x\}}{x}\le\frac{1}{x}$ for $x>0$

Hence $\lim\limits_{x\to\infty}\frac{\{x\}}{x}=0$

44. (a) : Statement-2 is clearly true.

$$\text{Now, } \lim_{x\to0}\left(\frac{\sin2x+a\sin x}{x^3}\right)=\lim_{x\to0}\left(\frac{\sin x}{x}\right)\left(\frac{2\cos x+a}{x^2}\right)$$

$$=\lim_{x\to0}\frac{2\cos x+a}{x^2}\text{ exists finitely}$$

Therefore $\lim\limits_{x\to0}(2\cos x+a)=0$

So $a=-2$

45. (a) : $\frac{b}{2}=\lim\limits_{x\to0}\frac{a\cos ax-1+\tan x}{2x}$

$$\Rightarrow\ b=\lim_{x\to0}\frac{a\cos ax-1}{x}+1\ \Rightarrow\ a=b=1$$

46. (c) : $\lim\limits_{x\to0}\frac{1+a\cos x-ax\sin x-b\cos x}{3x^2}=1$

implies $a-b=-1$ and in this case,

$$1=\lim_{x\to0}\frac{(1-\cos x)-ax\sin x}{3x^2}=\frac{1}{3}\left(\frac{1}{2}-a\right)$$

This gives $a=-\frac{5}{2}$ and $b=a+1=-\frac{3}{2}$

47. (c) : $L=\lim\limits_{x\to0}\frac{\frac{x}{\sqrt{a^2-x^2}}-x}{4x^3}=\lim\limits_{x\to0}\frac{-1+\frac{1}{\sqrt{a^2-x^2}}}{4x^2}$

$$\Rightarrow\ a=1\text{ and }L=\lim_{x\to0}\frac{-\sqrt{1-x^2}+1}{4x^2\sqrt{1-x^2}}=\frac{1}{8}$$

48. (a) : We have, $\lim\limits_{n\to\infty} f(x)$

$$=\lim_{n\to\infty}\frac{n(n-1)(n-2)\ldots(n-x+1)}{x!}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x}$$

$$=\frac{e^{-\lambda}\lambda^x}{x!}$$

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$$\therefore \sum_{x=0}^{\infty} xf(x) = \sum_{x=0}^{\infty} x.e^{-\lambda}\frac{\lambda^{x-1}\lambda}{(x-1)!x}$$

$$= \lambda e^{-\lambda}\sum_{x=1}^{\infty}\frac{\lambda^{x-1}}{(x-1)!} = e^{-\lambda}e^{\lambda}\lambda = \lambda$$

49. (b) : $\sum_{x=0}^{\infty} x^2 f(x) = \sum_{x=0}^{\infty} x(x-1)f(x) + \sum_{x=0}^{\infty} xf(x)$

$= \lambda^2 + \lambda$

50. (a) :

$$\sum_{x=0}^{\infty}(x-\lambda)^2 f(x) = \sum_{x=0}^{\infty} x^2 f(x) + \lambda^2\sum_{x=0}^{\infty} f(x) - 2\lambda\sum_{x=0}^{\infty} xf(x)$$

$= \lambda^2 + \lambda + \lambda^2 - 2\lambda^2 = \lambda$

51. A → (p); B → (r); C → (s); D → (q)

(A) $f(x) = \begin{cases} x, & x > 1 \\ 0, & x = 1 \\ -x, & x < 1 \end{cases}$. So $\lim_{x\to 1} f(x)$ does not exist.

(B) $\lim_{x\to 0}\frac{\sin(\sin(\tan(x^2/2)))}{\sin(\tan x^2/2)}$

$$\times\frac{\sin(\tan x^2/2)}{\tan(x^2/2)}\times\frac{\tan(x^2/2)}{x^2/2}\times\frac{x^2/2}{\log\cos 3x}$$

$$= \frac{1}{2}\lim_{x\to 0}\frac{2x}{-3\tan 3x} = \frac{-1}{9}$$

(C) $\lim_{x\to 0}\frac{\sqrt[3]{1+\tan^{-1}3x} - \sqrt[3]{1-\sin^{-1}3x}}{\sqrt{1-\sin^{-1}2x} - \sqrt{1+\tan^{-1}2x}}$

$$= \lim_{x\to 0}\frac{\left(1+\frac{1}{3}\tan^{-1}3x+.....\right)-\left(1-\frac{1}{3}\sin^{-1}3x+....\right)}{\left(1-\frac{1}{2}\sin^{-1}2x+.....\right)-\left(1+\frac{1}{2}\tan^{-1}2x+.....\right)}$$

$$= \lim_{x\to 0}\frac{\frac{1}{3}\frac{\tan^{-1}3x}{x}+\frac{1}{3}\frac{\sin^{-1}3x}{x}+\frac{1}{x}(\text{higher power of } \sin^{-1}3x \text{ and } \tan^{-1}3x)}{-\frac{1}{2}\frac{\sin^{-1}2x}{x}-\frac{1}{2}\frac{\tan^{-1}2x}{x}+\frac{1}{x}(\text{higher power of } \sin^{-1}2x \text{ and } \tan^{-1}2x)}$$

$$= \frac{1+1}{-1-1} = -1$$

(D) $\lim_{x\to 0^-}\frac{e^{1/x}-1}{e^{1/x}+1} = -1$ and $\lim_{x\to 0^+}\frac{e^{1/x}-1}{e^{1/x}+1} = 1$.

52. A → (r); B → (s); C → (q, r); D → (p)

(A) By L'Hospital's rule, we have

$$\lim_{x\to 3}\frac{(x^3+27)\frac{1}{x-2}+\log(x-2)3x^2}{2x} = \frac{54}{6} = 9$$

(B) Let $L = \lim_{x\to 0}\left(\frac{e^x-1}{x}\right)^{\left(\frac{x}{x+1-e^x}\right)} = \lim_{x\to 0}\left(\frac{e^x-1}{x}\right)^{\left(\frac{1}{1-\frac{e^x-1}{x}}\right)}$

$\therefore\ L = \lim_{t\to 1}(t)^{\frac{1}{1-t}}$ $[1^{\infty}$ form]

Taking logarithm,

$$\log L = \lim_{t\to 1}\frac{1}{1-t}\log t \quad \left(\frac{0}{0}\text{form}\right)$$

$$= \lim_{t\to 1}\frac{\frac{1}{t}}{-1} \text{ (By L'Hospital's rule)} = -1 \Rightarrow L = e^{-1}$$

(C) We know that $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + ...$

and $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - ....$

Let $L = \lim_{x\to 0}\frac{ax + x\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}-....\right)-b\left(x-\frac{x^3}{3!}+....\right)}{x^3}$

$$= \lim_{x\to 0}(a-b+1)\times\frac{1}{x^2}+\left(\frac{b}{3!}-\frac{1}{2!}\right) + \text{terms containing } x$$

As $L = 1$, so we must have

$a - b + 1 = 0$ and $\frac{b}{3!} - \frac{1}{2!} = 1 \Rightarrow b = 9$ and $a = 8$

(D) Let

$$L = \lim_{x\to 0}\frac{-f(x)+3f(2x)-3f(3x)+f(4x)}{x^3}\left(\frac{0}{0}\text{form}\right)$$

$$= \lim_{x\to 0}\frac{-f'(x)+6f'(2x)-9f'(3x)+4f'(4x)}{3x^2}$$

$$= \lim_{x\to 0}\frac{-f''(x)+12f''(2x)-27f''(3x)+16f''(4x)}{6x} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \lim_{x\to 0}\frac{-f'''(x)+24f'''(2x)-81f'''(3x)+64f'''(4x)}{6} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$= \frac{6.f'''(0)}{6} = f'''(0)$$

But $L = 12 \Rightarrow f'''(0) = 12$.

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CBSE warm-up!

CLASS-XI

Chapterwise Practice questions for CBSE Exams as per the reduced syllabus and marking scheme issued by CBSE for the academic session 2020-21.

Series 6

Straight Lines, Conic Sections and Introduction to Three Dimensional Geometry

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(1) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
(2) Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
(3) Both Part A and Part B have internal choices.

Part – A:

(1) It consists of two sections-I and II.
(2) Section I comprises of 16 very short answer type questions.
(3) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part – B:

(1) It consists of three sections-III, IV and V.
(2) Section III comprises of 10 questions of 2 marks each.
(3) Section IV comprises of 7 questions of 3 marks each.
(4) Section V comprises of 3 questions of 5 marks each.
(5) Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART A

SECTION - I

All questions are compulsory. In case of internal choices attempt any one.

1. Find the slope of line making inclination of 60° with positive direction of x-axis.

   OR

   If the slope of the line joining the points (3, 4) and $(-2, a)$ is equal to $-\frac{2}{5}$, then find the value of a.

2. What are the possible slopes of a line which makes equal angle with both axes?

3. Find the slope of a line joining two points $A(1, 2)$ and $B(3,-4)$.

   OR

   Find the equation of line through (–2, 3) with slope – 4.

4. Find the equation of the line through the points (1, –1) and (3, 5).

5. Find the equation of the line, where length of the perpendicular segment from the origin to the line is 4 and the inclination of the perpendicular segment with the positive direction of x-axis is 30°.

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OR

Find the equation of the line parallel to the y-axis and 3 units to the right of it.

6. Convert the equation $3x + 4y - 24 = 0$ into intercept form.

7. Find the value of k for which the line $(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$ is parallel to the x-axis.

OR

Find the distance of point $(3, -5)$ from the line $3x - 4y - 26 = 0$.

8. Find the distance between the parallel lines $y = mx + c$ and $y = mx + c'$.

9. Find the equation of the circle which passes through the point $(4, 5)$ and has its centre at $(2, 2)$.

OR

Write the equation of circle with centre $(-a, -b)$ and radius $\sqrt{a^2 + b^2}$.

10. Find the equation of the directrix of $(x - 1)^2 = 2(y - 2)$.

11. If the distance of the focus of a parabola from its directrix is 4, then find the length of the latus rectum.

12. Find the equation of the ellipse with foci at $(\pm 5, 0)$ and $x = \frac{36}{5}$ as one of the directices.

13. Find the eccentricity of the ellipse $12x^2 + 7y^2 = 84$.

14. What is the eccentricity of hyperbola whose vertices and foci are $(\pm 2, 0)$ and $(\pm 3, 0)$ respectively?

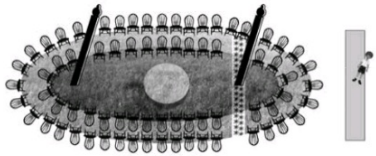
15. What does the equation $\frac{x^2}{14 - a} + \frac{y^2}{9 - a} = 1$ represent.

16. What is the distance of the point $(3, 4, 5)$ from the YZ plane?

SECTION - II

Both the case study based questions are compulsory. Attempt any 4 sub-parts from each question 17 and 18. Each question carries 1 mark.

17. Amit is standing on a line $\sqrt{3}x = 49$ outside of an open auditorium in the shape of ellipse. There are two poles each at $(\pm\sqrt{3}, 0)$, which are its foci. The line, where Amit is standing is one of the directrix of the auditorium.



Based on the above information, answer the following questions.

(i) The eccentricity of the auditorium is

(a) $\frac{7}{\sqrt{3}}$ (b) $7\sqrt{3}$ (c) $\frac{\sqrt{3}}{7}$ (d) $\frac{49}{\sqrt{3}}$

(ii) The equation of ellipse is

(a) $\frac{x^2}{2} + \frac{y^2}{1} = 7$ (b) $x^2 + y^2 = 49$

(c) $\frac{x^2}{49} + \frac{y^2}{12} = 1$ (d) $\frac{x^2}{49} + \frac{y^2}{46} = 1$

(iii) If the line between two vertices of the auditorium is arranged with flowers, then find the length of floral line passing through focus $(\sqrt{3}, 0)$ and parallel line on which Amit is standing.

(a) $\frac{92}{7}$ (b) $\frac{49}{3}$

(c) $\frac{72}{\sqrt{3}}$ (d) None of these

(iv) Length of Major axis of the auditorium is

(a) 24 units (b) 14 units

(c) 25 units (d) 15 units

(v) Find the equation of circular stage at centre of auditorium of radius 3.

(a) $x^2 + y^2 = 7$ (b) $x^2 + y^2 = \sqrt{7}$

(c) $x^2 + y^2 = 9$ (d) $x^2 + y^2 = 3$

18. A rectangular parallelopiped shaped jewellery box with one vertex $(3, 5, 6)$ is placed in the first octant with one vertex at origin and edges of the jewellery box are along x, y and z axis.

Based on the above information, answer the following questions.

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(i) If L, M, N be the feet of perpendiculars drawn from P on yz plane, zx plane and xy plane, then L, M, N are respectively

(a) (3, 0, 6), (0, 5, 6), (3, 5, 0)
(b) (3, 5, 0), (3, 0, 6), (0, 5, 6)
(c) (0, 5, 6) ; (3, 0, 6) ; (3, 5, 0)
(d) (3, 0, 6) ; (3, 5, 0) ; (0, 5, 6)

(ii) Sum of the length of edges of the jewellery box are

(a) 14 units (b) 5 units
(c) 6 units (d) 4 units

(iii) If any point $(x, 2, 3)$ is at a distance of $\sqrt{43}$ units from point P, then find the value of x.

(a) 8 (b) -2
(c) both (a) and (b) (d) None of these

(iv) If a point A divides $(-2, 2, 3)$ and point P into ratio $2 : 1$, then find the co-ordinates of point A.

(a) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{5}\right)$ (b) $\left(\frac{4}{3}, \frac{1}{5}, \frac{1}{3}\right)$
(c) $\left(\frac{2}{3}, \frac{4}{3}, 12\right)$ (d) $\left(\frac{4}{3}, 4, 5\right)$

(v) Find the co-ordinates of the point T which is three fifth of the way from P and L.

(a) (3, 5, 4) (b) (2, 5, 6)
(c) (6/5, 5, 6) (d) (1, 5, 6)

PART B

SECTION-III

All questions are compulsory. In case of internal choices attempt any one.

19. Find the equation of the curve formed by the set of all points whose distance from the points (3, 4, –5) and (–2, 1, 4) are equal.

20. Show that the points $A(0, 7, 10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$ form an isosceles right-angled triangle.

OR

Find the point on x-axis which is equidistant from the point $A(3, 2, 2)$ and $B(5, 5, 4)$.

21. Line through the points (–2, 6) and (4, 8) is perpendicular to the line through the points (8, 12) and $(x, 24)$. Find the value of x.

22. Find the slope of a line, which passes through the origin and the mid-point of the line segment joining the points (0, –4) and (8, 0).

23. Prove that the line $5x - 2y - 1 = 0$ is mid parallel to the lines $5x - 2y - 9 = 0$ and $5x - 2y + 7 = 0$.

OR

Which of the line $2x + 7y - 9 = 0$ and $4x - 4y + 11 = 0$ is farther from the point (2, 3)?

24. Find the equation of a circle which touches both the axes and the line $3x - 4y + 8 = 0$ and lies in the third quadrant.

25. If one end of a diameter of the circle $x^2 + y^2 - 4x - 6y + 11 = 0$ is (3, 4), then find the coordinate of the other end of the diameter.

26. Find the equation of the ellipse whose axis are along the coordinate axis, vertices are $(0, \pm 10)$ and eccentricity, $e = \frac{4}{5}$.

27. Find the equation of the ellipse whose vertices are $(\pm 13, 0)$ and foci are $(\pm 5, 0)$.

28. Find the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, which passes through the points $(3,0)$ and $(3\sqrt{2}, 2)$.

OR

Find the equation of the hyperbola whose foci is $(0, \pm 13)$ and length of conjugate axis is of 24 units.

SECTION-IV

All questions are compulsory. In case of internal choices attempt any one.

29. Find the equations of the line whose sum and product of intercepts on the co-ordinate axes are –1 and –12 respectively.

30. A line through the point $A(2,0)$ which makes an angle of 30° with the positive direction of x-axis is rotated about A in clockwise direction through an angle 15°. Find the equation of the straight line in the new position.

31. Find the equation of the set of points P such that $PA^2 + PB^2 = k^2$, where A and B are the points (3, 4, 5) and (–1, 3, –7) respectively.

OR

In a room, a globe is hanged in the middle of the room, using a string. Globe keeps on rotating depicting the history of 'STRUGGLE FOR INDEPENDENCE'. If globe is represented by the point (3, 4, 5). What is its distance from one corner of a room, assumed as origin?

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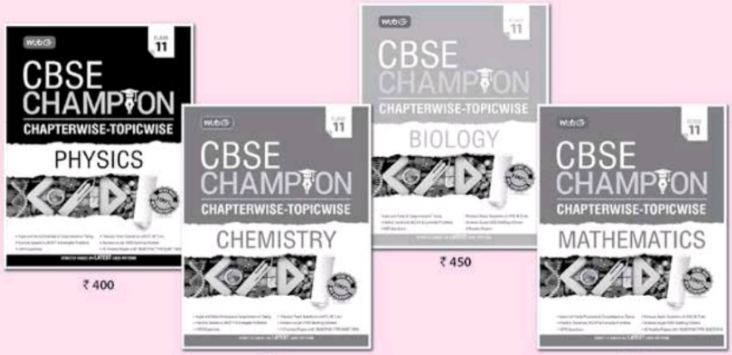

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CONCEPT MAP

STRAIGHT LINES

Class XI

General Equation

$ax + by + c = 0, |a| + |b| \neq 0$

Slope $= m = \tan\theta = -\frac{a}{b}$

x-intercept $= \frac{-c}{a}$, y-intercept $= \frac{-c}{b}$

Family of Lines

Let $L_1 = a_1x + b_1y + c_1 = 0$ and
$L_2 = a_2x + b_2y + c_2 = 0$ be two lines then,
$L_1 + \lambda L_2 = 0$ or $L_2 + \lambda L_1 = 0$ gives family of lines passing through the point of intersection of L_1 and L_2.

Concurrent lines :

- If $L_r = a_r x + b_r y + c_r = 0$ (where $r = 1, 2, 3$)
  then $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$
- $\lambda_1 L_1 + \lambda_2 L_2 + \lambda_3 L_3 = 0$, where $\lambda_1, \lambda_2, \lambda_3 \in R$, not all zero's.

Perpendicular Distance

- Distance of a point (x_1, y_1) from line $ax + by + c = 0$ is given by $\left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$.
- Distance between two parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is given by $\left|\frac{c_1 - c_2}{\sqrt{a^2 + b^2}}\right|$

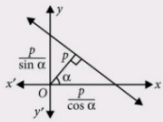
A Line and Two Points

$L : ax + by + c = 0$, $A(x_1, y_1)$ and $B(x_2, y_2)$.

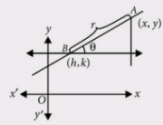
- A and B are on the same side of L if
  $$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} > 0$$
- A and B are on the opposite sides of L if
  $$\frac{ax_1 + by_1 + c}{ax_2 + by_2 + c} < 0$$
- AB is divided by L in the ratio
  $$\frac{-(ax_1 + by_1 + c)}{(ax_2 + by_2 + c)}$$

Standard Forms

- Slope-intercept form : $y = mx + c$
  Here, m is the slope of line making an intercept c on y-axis.
- Point-slope form : $\frac{y - y_1}{x - x_1} = m$
  Here, m is the slope and (x_1, y_1) be given point on the line.
- Two point form : $\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$
  Here (x_1, y_1) & (x_2, y_2) are two given points on the line.
- Intercept form : $\frac{x}{a} + \frac{y}{b} = 1$
  Here a and b represents x and y intercepts respectively of the line.
- Determinant form : $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$, where (x_1, y_1) and (x_2, y_2) are any two points on the line.
- Perpendicular form (or) Normal form of $ax + by + c = 0$ is $x\cos\alpha + y\sin\alpha = p$ where,
  $\sin\alpha = \frac{b}{\sqrt{a^2 + b^2}}$,
  $\cos\alpha = \frac{a}{\sqrt{a^2 + b^2}}$, $p = \frac{-c}{\sqrt{a^2 + b^2}} > 0$



- Parametric form : $\frac{x - h}{\cos\theta} = \frac{y - k}{\sin\theta} = r$
  Where $(x, y) = (h \pm r\cos\theta, k \pm r\sin\theta)$



Angle Between Two Lines

- For $L_1 = a_1x + b_1y + c_1 = 0$ & $L_2 = a_2x + b_2y + c_2 = 0$, acute angle (θ) is given by
  $$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1m_2}\right|,$$
  where m_1 = slope of L_1, m_2 = slope of L_2
- L_1 & L_2 are $\perp$r iff $m_1m_2 = -1$
- L_1 and L_2 are parallel iff $m_1 = m_2$

Foot of Perpendicular from a Point

For a point (h, k) and line $ax + by + c = 0$

foot of $\perp$r is given by $\frac{x - h}{a} = \frac{y - k}{b} = \frac{-(ah + bk + c)}{a^2 + b^2}$

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FUNCTIONS

CONCEPT MAP

Class XII

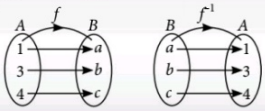
Definition

A relation $f: A \to B$ will be a function when every element of set A has only one image in set B.

Invertible Functions

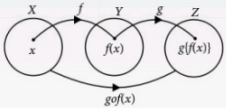
Let $f: A \to B$ be a bijection. Then inverse function is defined as $f^{-1}: B \to A$ *i.e.* $f^{-1}(b) = a \Leftrightarrow f(a) = b$.

Arrow diagram :



Composition of Functions

For two functions $f: X \to Y$ and $g: Y \to Z$, composition of functions is denoted by gof and defined as $gof(x) = g(f(x)) \ \forall \ x \in X$.



Properties

- Composition of functions is not commutative, *i.e.*, $fog \neq gof$ (in general).
- Composition of functions is associative *i.e.*, $fo(goh) = (fog)oh$
- Composition of two bijections is also a bijection.

Number of Functions

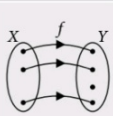
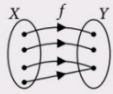
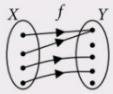
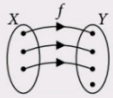
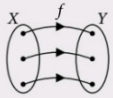
Let $f: A \to B$ such that $n(A) = m$, $n(B) = n$.

- The number of functions is n^m.
- The number of one-one functions are ${}^nP_m = \dfrac{n!}{(n-m)!}$, if $m \leq n$.
- The number of onto functions are

$$n^m - \binom{n}{1}(n-1)^m + \binom{n}{2}(n-2)^m - \binom{n}{3}(n-3)^m + \ldots, \text{ if } m \geq n.$$

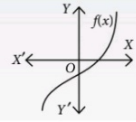
- The number of bijections is $n!$, if $m = n$.

Types of Functions

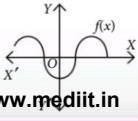
Type	Definition	Representation
One-one (Injective) Function	A function $f: X \to Y$ is one-one, if different elements of X have different images in Y under f. (i) $n(X) \leq n(Y)$ (ii) $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$	
Onto (Surjective) Function	A function $f: X \to Y$ is onto, if every element of Y is the image of atleast one element of X under f. (i) $n(X) \geq n(Y)$ (ii) Range = Co-domain	
Many-one Function	A function $f: X \to Y$ is many-one, if two or more than two elements of X have the same image in Y.	
Into Function	A function $f: X \to Y$ is into, if there exists at least one element in Y having no pre-image in X.	
Bijective Function	A function $f: X \to Y$ is bijective, if it is both one-one and onto *i.e.*, $n(X) = n(Y)$ and Range = Co-domain	

Analysis of Functions by Graphical Method

- If any line parallel to x axis intersects graph of $y = f(x)$ not more than once, then it is one-one.



- If $f'(x)$ is always strictly increasing or decreasing then it is one-one.
- If atleast one line parallel to the x axis intersects more than once or $f'(x) = 0$ over a continuous interval or changes its sign, then it is not one-one.



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32. Find the equation of the circle passing through (0, 0) and making intercepts a and b on the coordinate axes.

33. Find the equation of the parabola whose focus is the point (0, 0) and directrix is the straight line $3x - 4y + 2 = 0$.

34. Find the coordinates of the focus, equation of the directrix and length of the latus rectum of the parabola $x^2 = -16y$.

OR

Find the equation of the ellipse with focus at (1, 1) and eccentricity 1/2 and directrix $x - y + 3 = 0$. Also, find the equation of its major axis.

35. Find the foci, vertices, eccentricity and length of latus rectum of ellipse $4x^2 + 9y^2 = 36$.

SECTION - V

All questions are compulsory. In case of internal choices attempt any one.

36. The mid-points of the sides of a triangle are (1, 5, –1), (0, 4, –2) and (2, 3, 4). Find its vertices.

OR

$ABCD$ is a parallelogram whose 3 consecutive vertices are $A(1, 2, 3)$, $B(-1, -2, -1)$ and $C(2, 3, 2)$. Find the fourth vertex D. Check whether $ABCD$ is a rectangle or not.

37. Find the lengths of transverse axis, conjugate axis, eccentricity, coordinates of foci, vertices, length of the latus rectum and equations of the directrices for the hyperbola $3x^2 - 6y^2 = -18$.

OR

If e and e' be the eccentricities of a hyperbola and its conjugate, then prove that $\frac{1}{e^2} + \frac{1}{e'^2} = 1$.

38. Prove that product of the lengths of the perpendiculars drawn from the points $\left(\sqrt{a^2 - b^2}, 0\right)$ and $\left(-\sqrt{a^2 - b^2}, 0\right)$ to the line $\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$ is b^2.

OR

Prove that the diagonals of the parallelogram formed by the lines $\frac{x}{a} + \frac{y}{b} = 1$, $\frac{x}{b} + \frac{y}{a} = 1$, $\frac{x}{a} + \frac{y}{b} = 2$ and $\frac{x}{b} + \frac{y}{a} = 2$ are at right angles.

SOLUTIONS

1. Let m be the slope of the line, then
$m = \tan 60° = \sqrt{3}$

OR

Given, slope of the line $= -\frac{2}{5}$

$\Rightarrow \quad \frac{a-4}{-2-3} = -\frac{2}{5} \Rightarrow \frac{a-4}{-5} = -\frac{2}{5} \Rightarrow a - 4 = 2 \Rightarrow a = 6$

2. The possible slopes of a line that makes equal angle with both axes are $m = 1$ and $m = -1$.

3. Slope of line joining points $A(1, 2)$ and $B(3, -4)$ is given by

$$m = \frac{-4-2}{3-1} = \frac{-6}{2} = -3$$

OR

The equation of line having slope – 4 and passes through the point (–2, 3) is given by
$y - 3 = (-4)[x - (-2)]$
$\Rightarrow \quad y - 3 = -4(x + 2) = -4x - 8 \quad \Rightarrow \quad 4x + y + 5 = 0$

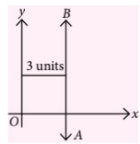
4. Using two-point form, equation of line is

$$y - (-1) = \frac{5-(-1)}{3-1}(x-1) \Rightarrow 3x - y - 4 = 0$$

5. Required equation of the line in normal form is
$x \cos 30° + y \sin 30° = 4$

$\Rightarrow \quad x\frac{\sqrt{3}}{2} + y\frac{1}{2} = 4 \Rightarrow \sqrt{3}x + y = 8$

OR



Since the line is at a distance of 3 units to the right of y-axis,
$\therefore$ Equation of required line is $x = 3$.

6. The given equation is
$3x + 4y - 24 = 0$
Dividing both sides by 24, we get

$$\frac{3x}{24} + \frac{4y}{24} = 1 \Rightarrow \frac{x}{8} + \frac{y}{6} = 1$$

7. Given equation of the line is
$(k - 3)x - (4 - k^2)y + k^2 - 7k + 6 = 0$
Since, the given line is parallel to x-axis.
$\therefore$ Coeff. of $x = 0 \Rightarrow k - 3 = 0 \Rightarrow k = 3$

OR

Let d be the required distance, then

$$d = \frac{|3\times 3 - 4\times(-5) - 26|}{\sqrt{(3)^2 + (-4)^2}} = \frac{|9 + 20 - 26|}{\sqrt{9+16}} = \frac{3}{5} \text{ unit}$$

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8. The given lines are

$mx - y + c = 0$ ...(i)

and $mx - y + c' = 0$ ...(ii)

Let $P(x_1, y_1)$ be any point on (i), so that

$mx_1 - y_1 + c = 0$ or $mx_1 - y_1 = -c$ ...(iii)

Distance between parallel lines (i) and (ii)

= Distance of $P(x_1, y_1)$ from (ii)

$$= \left|\frac{mx_1 - y_1 + c'}{\sqrt{m^2+1}}\right| = \left|\frac{-c + c'}{\sqrt{m^2+1}}\right| \text{ units} \quad \text{[Using (iii)]}$$

9. As the circle is passing through the point (4, 5) and its centre is (2, 2), so its radius is $\sqrt{(4-2)^2+(5-2)^2} = \sqrt{13}$

$\therefore$ Required equation of circle is $(x-2)^2 + (y-2)^2 = 13$.

OR

The equation of circle with centre $(-a, -b)$ and radius $\sqrt{a^2+b^2}$ is given by

$$[x-(-a)]^2 + [y-(-b)]^2 = \left(\sqrt{a^2+b^2}\right)^2$$

$\Rightarrow (x+a)^2 + (y+b)^2 = a^2 + b^2$

$\Rightarrow x^2 + a^2 + 2ax + y^2 + b^2 + 2by = a^2 + b^2$

$\Rightarrow x^2 + y^2 + 2ax + 2by = 0$

10. Equation of directrix of parabola $x^2 = 4ay$ is $y = -a$.

$\therefore \quad y - 2 = -\frac{1}{2} \Rightarrow 2(y-2) = -1 \Rightarrow 2y - 3 = 0$

11. Since, distance of the focus of a parabola from its directrix is $2a$.

$\therefore$ Given, $2a = 4 \Rightarrow a = 2$

Now, length of latus rectum $= 4a = 8$

12. We have, $ae = 5$, $\frac{a}{e} = \frac{36}{5}$, which gives $a^2 = 36$ or $a = 6$. Therefore, $e = \frac{5}{6}$.

Now, $b = a\sqrt{1-e^2} = 6\sqrt{1-\frac{25}{36}} = \sqrt{11}$.

Thus, the equation of the ellipse is $\frac{x^2}{36} + \frac{y^2}{11} = 1$.

13. Given equation of ellipse is $12x^2 + 7y^2 = 84$

$$\Rightarrow \frac{x^2}{\left(\frac{84}{12}\right)} + \frac{y^2}{\left(\frac{84}{7}\right)} = 1 \Rightarrow \frac{x^2}{7} + \frac{y^2}{12} = 1$$

$\therefore \quad a^2 = 7, b^2 = 12$

Now, $e = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{7}{12}} = \sqrt{\frac{5}{12}}$

14. Vertices are $(\pm 2, 0)$ and foci are $(\pm 3, 0)$

$\Rightarrow \quad a = 2, ae = 3 \therefore \quad e = \frac{3}{2}$

15. We have, $\frac{x^2}{14-a} + \frac{y^2}{9-a} = 1$

The equation will represent an ellipse if $14 - a > 0$ and $9 - a > 0$

$\Rightarrow \quad a < 14$ and $a < 9 \Rightarrow a < 9$

The given equation represent a hyperbola if $14 - a > 0$ and $9 - a < 0$

$\Rightarrow \quad a < 14$ and $a > 9 \Rightarrow 9 < a < 14$

16. The coordinates of foot of perpendicular from the point (3, 4, 5) on YZ plane are (0, 4, 5).

$\therefore$ Required distance $= \sqrt{(0-3)^2 + (4-4)^2 + (5-5)^2}$

$= \sqrt{9+0+0} = 3$ units

17. **(i) (c) :** Here, foci $\equiv (\pm ae, 0) = \left(\pm\sqrt{3}, 0\right)$

$\Rightarrow \quad ae = \sqrt{3}$ ...(i)

Also, equation of directrix is $\sqrt{3}x - 49 = 0$.

$\Rightarrow \quad x = \frac{49}{\sqrt{3}} \quad \therefore \quad \frac{a}{e} = \frac{49}{\sqrt{3}}$ ...(ii)

From (i) and (ii), we get

$$ae.\frac{a}{e} = \sqrt{3} \times \frac{49}{\sqrt{3}} = 49 \Rightarrow a^2 = 49 \Rightarrow a = 7$$

From (i), $e = \frac{\sqrt{3}}{7}$

(ii) (d) : As, $e^2 = \frac{a^2 - b^2}{a^2}$

$$\Rightarrow \left(\frac{\sqrt{3}}{7}\right)^2 = \frac{7^2 - b^2}{7^2} \Rightarrow 3 = 49 - b^2$$

$\Rightarrow \quad b^2 = 46$

$\therefore \quad$ Equation of ellipse is $\frac{x^2}{49} + \frac{y^2}{46} = 1$

(iii) (a) : Here, floral line passing through focus and parallel to directrix is the latus rectum.

$\therefore \quad$ Length of latus rectum $= \frac{2b^2}{a} = \frac{2 \times 46}{7} = \frac{92}{7}$

(iv) (b) : Length of major axis $= 2a = 2 \times 7 = 14$ units

(v) (c) : Here, centre of the elliptical auditorium is (0, 0) and radius of circle is 3.

$\therefore \quad$ Equation of circular stage $= (x-0)^2 + (y-0)^2 = 3^2$

$\Rightarrow \quad x^2 + y^2 = 9$

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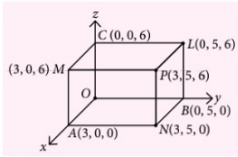
18. (i) (c) :



Since, L, M and N be the feet of perpendiculars drawn from P on yz-plane, zx-plane, and xy-plane respectively.

$\therefore$ L is (0, 5, 6), M is (3, 0, 6) and N is (3, 5, 0).

(ii) (a) : Here, PL, PM, PN are edges of jewellery box.

$\therefore$ $|PL| = |OA| = 3$, $|PM| = |OB| = 5$

and $|PN| = |OC| = 6$

$\therefore$ Required sum = 3 + 5 + 6 = 14 units

(iii) (c) : According to given condition,

$$\sqrt{(3-x)^2+(5-2)^2+(6-3)^2} = \sqrt{43}$$

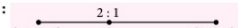
$\Rightarrow$ $(3-x)^2 + 9 + 9 = 43$

$\Rightarrow$ $(3-x)^2 = 25$

$\Rightarrow$ $3 - x = \pm 5$

$\Rightarrow$ $x = 8$ or -2

(iv) (d) :



Co-ordinates of A are $\left(\frac{6-2}{3}, \frac{10+2}{3}, \frac{12+3}{3}\right)$ *i.e.*, $\left(\frac{4}{3}, 4, 5\right)$.

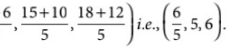
(v) (c) : According to question,

$$PT = \frac{3}{5}PL \Rightarrow 5PT = 3(PT + TL)$$

$\Rightarrow$ $5PT = 3PT + 3TL$

$\Rightarrow$ $2PT = 3TL \Rightarrow \frac{PT}{TL} = \frac{3}{2}$

$\therefore$ T divides PL into ratio 3 : 2.



Thus co-ordinates of

T are $\left(\frac{0+6}{5}, \frac{15+10}{5}, \frac{18+12}{5}\right)$ *i.e.*, $\left(\frac{6}{5}, 5, 6\right)$.

19. Let $P(x, y, z)$ be any point on the given curve, and $A(3, 4, -5)$ and $B(-2, 1, 4)$ be the given points.

Then, $PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow (x-3)^2 + (y-4)^2 + (z+5)^2 = (x+2)^2 + (y-1)^2 + (z-4)^2$

$\Rightarrow$ $10x + 6y - 18z - 29 = 0$.

Hence, the required equation of curve is

$10x + 6y - 18z - 29 = 0$

20. Given, $A(0, 7, 10)$, $B(-1, 6, 6)$ and $C(-4, 9, 6)$.

$\therefore$ Using distance formula, we have

$$AB = \sqrt{(-1-0)^2+(6-7)^2+(6-10)^2} = \sqrt{18} = 3\sqrt{2}$$

$$BC = \sqrt{(-4+1)^2+(9-6)^2+(6-6)^2} = \sqrt{18} = 3\sqrt{2}$$

$$AC = \sqrt{(-4-0)^2+(9-7)^2+(6-10)^2} = \sqrt{36} = 6$$

Clearly, $AB = BC$ and $AB^2 + BC^2 = 18 + 18 = 36 = AC^2$.

Hence, triangle ABC is an isosceles right-angled triangle.

OR

The point on the x-axis is of the form $P(x, 0, 0)$. Since the points $A(3, 2, 2)$ and $B(5, 5, 4)$ are equidistant from P.

$\therefore$ $PA = PB \Rightarrow PA^2 = PB^2$

$\Rightarrow$ $(x-3)^2 + (0-2)^2 + (0-2)^2 = (x-5)^2 + (0-5)^2 + (0-4)^2$

$\Rightarrow$ $4x = 25 + 25 + 16 - 17 \Rightarrow x = \frac{49}{4}$

Thus, the point $P\left(\frac{49}{4}, 0, 0\right)$, on the x-axis is equidistant from A and B.

21. As the line through $(-2, 6)$ and $(4, 8)$ is perpendicular to the line through $(8, 12)$ and $(x, 24)$

$\because$ Product of the slopes of two lines = -1

$\Rightarrow$ $\frac{8-6}{4+2} \times \frac{24-12}{x-8} = -1 \Rightarrow \frac{2}{6} \times \frac{12}{x-8} = -1$

$\Rightarrow$ $x - 8 = -4 \Rightarrow x = 4$

22. Mid-point of the given line

$$= \left(\frac{0+8}{2}, \frac{-4+0}{2}\right) = (4, -2)$$

Let m be the slope of the line containing the points (0,0) and (4,–2), then

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-0}{4-0} = \frac{-2}{4} = \frac{-1}{2}$$

23. The given equations of lines are

$5x - 2y - 1 = 0$ ...(i)

$5x - 2y - 9 = 0$ ...(ii)

and $5x - 2y + 7 = 0$ ...(iii)

Slope of each line = $\frac{5}{2}$

The line (i) is parallel to lines (ii) and (iii).

Distance between (i) and (ii)

$$= \frac{|-1-(-9)|}{\sqrt{5^2+(-2)^2}} = \frac{8}{\sqrt{29}} \text{ units}$$

Distance between (i) and (iii)

$$= \frac{|-1-7|}{\sqrt{5^2+(-2)^2}} = \frac{8}{\sqrt{29}} \text{ units}$$

The line (i) is equidistant from lines (ii) and (iii). Hence, the line (i) is mid-parallel to the lines (ii) and (iii).

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OR

Given equations of lines are

$2x + 7y - 9 = 0$ ...(i)

and $4x - 4y + 11 = 0$ ...(ii)

Length of perpendicular from (2, 3) from line (i) is

$$\left|\frac{2(2)+7(3)-9}{\sqrt{2^2+7^2}}\right| = \left|\frac{4+21-9}{\sqrt{4+49}}\right| = \frac{16}{\sqrt{53}} \text{ units}$$

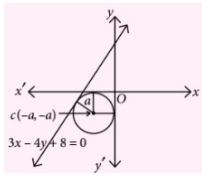
Length of perpendicular from (2, 3) from line (ii) is

$$\left|\frac{4(2)-4(3)+11}{\sqrt{4^2+4^2}}\right| = \left|\frac{8-12+11}{\sqrt{16+16}}\right| = \frac{7}{4\sqrt{2}} \text{ units}$$

Thus, $2x + 7y - 9 = 0$ is farther from the point (2, 3).

24. Let a be the radius of the circle. Since circle touches both the axes and lies in third quadrant.

$\therefore$ $(-a, -a)$ will be centre of circle and perpendicular distance from the centre to the given line gives the radius of the circle.



Since, $d = \left|\frac{-3a+4(a)+8}{\sqrt{9+16}}\right|$

$= \left|\frac{a+8}{5}\right| = \pm\left(\frac{a+8}{5}\right)$ $\therefore$ $a = \frac{a+8}{5}$ and $a = \frac{-a-8}{5}$

$\Rightarrow$ $a = 2$ and $a = -4/3$

But $a \neq -4/3 \Rightarrow a = 2$

Hence, required equation of circle is

$(x + 2)^2 + (y + 2)^2 = 2^2 \Rightarrow x^2 + y^2 + 4x + 4y + 4 = 0$

25. Given equation of the circle is

$x^2 + y^2 - 4x - 6y + 11 = 0$ ...(i)

Compare (i) with general equation

$x^2 + y^2 + 2gx + 2fy + c = 0$ of circle, we get

$2g = -4$ and $2f = -6$

$\Rightarrow$ Centre $\equiv (-g, -f) = (2,3)$

Let the co-ordinates of the other end of diameter of the circle are (x, y). Now, (2, 3) is the mid-point of diameter.

$\therefore$ $2 = \frac{3+x}{2} \Rightarrow x = 1$ and $3 = \frac{4+y}{2} \Rightarrow y = 2$

Hence, co-ordinates of the other end of the diameter of circle are (1,2).

26. Let the equation of the required ellipse be

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad ...(i)$$

Coordinates of the vertices are $(0, \pm b)$.

$\therefore$ $b = 10$

Now, $a^2 = b^2(1 - e^2) \Rightarrow a^2 = 100\left(1 - \frac{16}{25}\right) = 36$

Substituting the values of a^2 and b^2 in (i), we obtain $\frac{x^2}{36} + \frac{y^2}{100} = 1$ as the equation of the required ellipse.

27. Since, vertices of the ellipse are $(\pm 13, 0)$, and foci are $(\pm 5, 0)$.

$\therefore$ Major axes is along x-axis.

Let equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ ...(i)

foci $\equiv (\pm 5, 0) = (\pm c, 0) \Rightarrow c = 5 \Rightarrow c^2 = 25$

$\Rightarrow$ $a^2 - b^2 = 25$ ...(ii)

Vertices $\equiv (\pm 13, 0) = (\pm a, 0) \Rightarrow a = 13$ ...(iii)

From (ii) and (iii), $b^2 = 169 - 25 = 144 \Rightarrow b = 12$

Putting $a = 13$, $b = 12$ in (i), we get

$$\frac{x^2}{169} + \frac{y^2}{144} = 1.$$

28. Since, the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is passing through the points (3, 0) and $(3\sqrt{2}, 2)$, so we get $a^2 = 9$ and $b^2 = 4$.

Also, we know that $b^2 = a^2(e^2 - 1)$

$\Rightarrow$ $4 = 9(e^2 - 1) \Rightarrow e^2 = \frac{13}{9} \Rightarrow e = \frac{\sqrt{13}}{3}$

OR

Since, foci of hyperbola is $(0, \pm 13)$

Let equation of hyperbola is $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$ ...(i)

Length of conjugate axis $= 2b = 24$

$\Rightarrow$ $b = 12 \Rightarrow b^2 = 144$

Foci $\equiv (0, +13) = (0, \pm c) \Rightarrow c = 13 \Rightarrow c^2 = 169$

We know that $a^2 + b^2 = c^2$

$\Rightarrow$ $a^2 + 144 = 169 \Rightarrow a = 5$

$\therefore$ Required equation of hyperbola is

$$\frac{y^2}{25} - \frac{x^2}{144} = 1$$

29. Let the equation in intercept form be $\frac{x}{a} + \frac{y}{b} = 1$

According to question, $a + b = -1$ ...(i)

and $ab = -12$ ...(ii)

$\therefore$ $(a - b)^2 = (a + b)^2 - 4ab$

$= (-1)^2 - 4(-12) = 49 = 7^2$

$\Rightarrow$ $a - b = 7$ ...(iii)

and $a - b = -7$ ...(iv)

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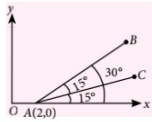
Solving (i) and (iii), we get
$a = 3$ and $b = -4$
$\therefore$ The required equation is
$\frac{x}{3} + \frac{y}{-4} = 1$
$\Rightarrow \quad 4x - 3y - 12 = 0$
Solving (i) and (iv), we get
$a = -4$ and $b = 3$
$\therefore$ The required equation is
$-\frac{x}{4} + \frac{y}{3} = 1 \Rightarrow 3x - 4y + 12 = 0$

30. Given, $A \equiv (2, 0)$. AB is the initial position of the line and AC is its new position.



Given, $\angle BAX = 30°$
and $\angle BAC = 15°$
$\therefore \quad \angle CAX = 30° - 15° = 15°$
Now, slope of line AC
$= \tan 15° = 2 - \sqrt{3}$
Now equation of line AC will be
$y - 0 = (2 - \sqrt{3})(x - 2)$
$\Rightarrow \quad y = (2 - \sqrt{3})x - 4 + 2\sqrt{3}$
$\Rightarrow \quad (2 - \sqrt{3})\, x - y - 4 + 2\sqrt{3} = 0$

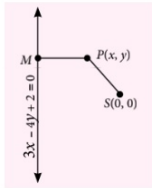
31. Let point P be (x, y, z).
Given points are $A(3, 4, 5)$ and $B(-1, 3, -7)$.
We have, $PA^2 + PB^2 = k^2$
$\Rightarrow \quad [(x-3)^2 + (y-4)^2 + (z-5)^2] + [(x+1)^2 + (y-3)^2 + (z+7)^2] = k^2$
$\Rightarrow \quad x^2 - 6x + 9 + y^2 - 8y + 16 + z^2 - 10z + 25 + x^2 + 2x + 1 + y^2 - 6y + 9 + z^2 + 14z + 49 = k^2$
$\Rightarrow \quad 2x^2 + 2y^2 + 2z^2 - 4x - 14y + 4z + 109 - k^2 = 0$, which is the required equation.

OR

Refer to answer 35, Page no. 270 of MTG CBSE Champion, Mathematics, Class 11

32. Given circle passes through origin. Let it meets axes at A and B, then AB is diameter and coordinates of end are $A(a, 0)$, $B(0, b)$.
$\therefore$ Coordinates of centre are $P\left(\frac{a}{2}, \frac{b}{2}\right)$ and radius is $\frac{1}{2}\sqrt{a^2 + b^2}$.
$\therefore$ Equation of circle is
$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \left(\frac{1}{2}\sqrt{a^2 + b^2}\right)^2$
$\Rightarrow \quad x^2 + \frac{a^2}{4} - ax + y^2 + \frac{b^2}{4} - by = \frac{1}{4}(a^2 + b^2)$
$\Rightarrow \quad x^2 + y^2 - ax - by = 0$ is the required equation.

33. Let $P(x, y)$ be any point on the parabola whose focus is $S(0, 0)$ and directrix is $3x - 4y + 2 = 0$. Draw PM perpendicular from P on the directrix. Then, by definition



$SP = PM$
$\Rightarrow \quad SP^2 = PM^2$
$\Rightarrow \quad (x-0)^2 + (y-0)^2 = \left|\frac{3x - 4y + 2}{\sqrt{3^2 + (-4)^2}}\right|^2$
$\Rightarrow \quad x^2 + y^2 = \frac{(3x - 4y + 2)^2}{25}$
$\Rightarrow \quad 25(x^2 + y^2) = (3x - 4y + 2)^2$
$\Rightarrow \quad 25x^2 + 25y^2 = 9x^2 + 16y^2 + 4 - 24xy - 16y + 12x$
$\Rightarrow \quad 16x^2 + 9y^2 + 24xy - 12x + 16y - 4 = 0$
This is the required equation of the parabola.

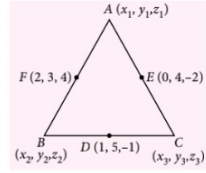
34. Consider the equation $x^2 = -16y$
Comparing with $x^2 = -4ay$, we get
$4a = 16 \Rightarrow a = 4$
$\therefore$ Coordinates of focus are $(0, -4)$
Equation of directrix is $y - a = 0 \Rightarrow y - 4 = 0$
Length of latus rectum $= |4a| = 16$

OR

Refer to answer 76, Page no. 255 of MTG CBSE Champion, Mathematics, Class 11

35. We have, $4x^2 + 9y^2 = 36$
$\Rightarrow \quad \frac{x^2}{9} + \frac{y^2}{4} = 1$, where $a = 3$, $b = 2$
Now, $c^2 = a^2 - b^2 = 9 - 4 = 5 \Rightarrow c = \sqrt{5}$
Foci $\equiv (\pm c, 0) = (\pm\sqrt{5}, 0)$, vertices $\equiv (\pm a, 0) = (\pm 3, 0)$
Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{5}}{3}$
Latus rectum $= \frac{2b^2}{a} = \frac{2 \times 4}{3} = \frac{8}{3}$

36. Let $A(x_1, y_1, z_1)$, $B(x_2, y_2, z_2)$ and $C(x_3, y_3, z_3)$ be the vertices of the given triangle and let $D(1,5,-1)$, $E(0,4,-2)$ and $F(2,3,4)$ be the mid-points of the sides BC, CA and AB respectively.



As, D is the mid-point of BC.

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$\therefore \quad \frac{x_2+x_3}{2}=1, \frac{y_2+y_3}{2}=5, \frac{z_2+z_3}{2}=-1$

$\Rightarrow \quad x_2+x_3=2, y_2+y_3=10, z_2+z_3=-2 \quad ...(i)$

As, E is the mid-point of CA.

$\therefore \quad \frac{x_1+x_3}{2}=0, \frac{y_1+y_3}{2}=4, \frac{z_1+z_3}{2}=-2$

$\Rightarrow \quad x_1+x_3=0, y_1+y_3=8, z_1+z_3=-4 \quad ...(ii)$

Again, F is the mid-point of AB.

$\therefore \quad \frac{x_1+x_2}{2}=2, \frac{y_1+y_2}{2}=3, \frac{z_1+z_2}{2}=4$

$\Rightarrow \quad x_1+x_2=4, y_1+y_2=6, z_1+z_2=8 \quad ...(iii)$

Adding first equations in (i), (ii) and (iii), we get,

$2(x_1+x_2+x_3)=6$

$\Rightarrow \quad x_1+x_2+x_3=3 \quad ...(iv)$

Solving first equations in (i), (ii) and (iii) with (iv), we get, $x_1=1, x_2=3, x_3=-1$

Adding second equations in (i), (ii) and (iii), we get,

$2(y_1+y_2+y_3)=10+8+6$

$\Rightarrow \quad y_1+y_2+y_3=12 \quad ...(v)$

Solving second equations in (i), (ii) and (iii) with (v), we get, $y_1=2, y_2=4, y_3=6$

Adding last equations in (i), (ii) and (iii), we get,

$2(z_1+z_2+z_3)=-2-4+8 \Rightarrow z_1+z_2+z_3=1 \quad ...(vi)$

Solving last equations in (i), (ii) and (iii) with (vi) we get, $z_1=3, z_2=5, z_3=-7$

Thus, the vertices of the triangle are $A(1,2,3)$, $B(3,4,5)$ and $C(-1, 6, -7)$.

OR

Refer to answer 74, Page no. 277 of MTG CBSE Champion, Mathematics, Class 11

37. The equation $3x^2-6y^2=-18$ can be written as $\frac{x^2}{6}-\frac{y^2}{3}=-1$, where $a^2=6$ and $b^2=3$

Length of the transverse axis $=2b=2\sqrt{3}$

Length of the conjugate axis $=2a=2\sqrt{6}$

Eccentricity, $e=\sqrt{1+\frac{a^2}{b^2}}=\sqrt{1+\frac{6}{3}}=\sqrt{3}$

Foci are $(0, \pm 3)$, Vertices are $(0, \pm\sqrt{3})$

Latus rectum $=\frac{2a^2}{b}=\frac{12}{\sqrt{3}}=4\sqrt{3}$

Equations of the directrices are $y=\pm b/e$ *i.e.*, $y=\pm 1$

OR

Let the equation of the hyperbola be

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1 \quad ...(i)$

Then, the equation of the hyperbola conjugate to (i) is

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=-1 \quad ...(ii)$

We have, e = Eccentricity of (i)

$$e=\sqrt{1+\left(\frac{\text{conjugate axis}}{\text{transverse axis}}\right)^2}$$

$$\Rightarrow \quad e=\sqrt{1+\left(\frac{2b}{2a}\right)^2} \Rightarrow e^2=1+\frac{b^2}{a^2} \Rightarrow e^2=\frac{a^2+b^2}{a^2} \quad ...(iii)$$

and, e' = Eccentricity of (ii)

$$=\sqrt{1+\left(\frac{\text{conjugate axis}}{\text{transverse axis}}\right)^2}$$

$$\Rightarrow \quad e'=\sqrt{1+\left(\frac{2a}{2b}\right)^2} \Rightarrow e'^2=1+\frac{a^2}{b^2} \Rightarrow e'^2=\frac{a^2+b^2}{b^2} \quad ...(iv)$$

From (iii) and (iv), we have

$$\frac{1}{e^2}+\frac{1}{e'^2}=\frac{a^2}{a^2+b^2}+\frac{b^2}{a^2+b^2}=\frac{a^2+b^2}{a^2+b^2} \Rightarrow \frac{1}{e^2}+\frac{1}{e'^2}=1$$

38. Let $\sqrt{a^2-b^2}=c$

$$\therefore \quad d_1=\frac{\left|\frac{c\cdot\cos\theta}{a}-1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}$$

$$\text{and } d_2=\frac{\left|\frac{-c\cdot\cos\theta}{a}-1\right|}{\sqrt{\left(\frac{\cos\theta}{a}\right)^2+\left(\frac{\sin\theta}{b}\right)^2}}$$

$$\therefore \quad d_1\times d_2=\frac{\left|\frac{c\cdot\cos\theta}{a}-1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}\cdot\frac{\left|\frac{-c\cdot\cos\theta}{a}-1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$$

$$=\frac{\left|\frac{c^2\cos^2\theta}{a^2}-1\right|\times a^2b^2}{b^2\cos^2\theta+a^2\sin^2\theta}=\frac{|(a^2-b^2)\cos^2\theta-a^2|\times b^2}{(b^2\cos^2\theta+a^2\sin^2\theta)}$$

$$=\frac{|a^2(1-\sin^2\theta)-b^2\cos^2\theta-a^2|}{(b^2\cos^2\theta+a^2\sin^2\theta)}\times b^2$$

$$=\frac{|-(a^2\sin^2\theta+b^2\cos^2\theta)|}{(b^2\cos^2\theta+a^2\sin^2\theta)}\times b^2$$

$$=\frac{(a^2\sin^2\theta+b^2\cos^2\theta)}{(a^2\sin^2\theta+b^2\cos^2\theta)}\times b^2=b^2$$

OR

Refer to answer 95, Page no. 230 of MTG CBSE Champion, Mathematics, Class 11

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MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80 | **Time Taken : 60 Min.**

Series 6 : Sequences and Series

Only One Option Correct Type

1. If three positive real numbers a, b, c are in A.P. such that $abc = 4$, then the minimum value of b is
   (a) $2^{1/3}$ (b) $2^{2/3}$
   (c) $2^{1/2}$ (d) $2^{3/2}$

2. If a, x, b are in A.P., a, y, b are in G.P. and a, z, b are in H.P. such that $x = 9z$ and $a > 0, b > 0$, then
   (a) $|y| = 3z$ and $x = 3|y|$
   (b) $y = 3|z|$ and $|x| = 3y$
   (c) $2y = x + z$
   (d) None of these

3. If a, b, c, are in A.P., then $\frac{a}{bc}, \frac{1}{c}, \frac{2}{b}$ will be in
   (a) A.P. (b) G.P.
   (c) H.P. (d) None of these

4. If $(1+x)(1+x^2)(1+x^4)\ldots(1+x^{128}) = \sum_{r=0}^{n} x^r$, then '$n$' is equal to
   (a) 256 (b) 255
   (c) 254 (d) None of these

5. If the first, fifth and last terms of an A.P. is l, m, p, respectively, and sum of the A.P. is $\frac{(l+p)(4p+m-5l)}{k(m-l)}$, then k is
   (a) 2 (b) 3 (c) 4 (d) 5

6. If a is the A.M. of b and c and two geometric means are G_1 and G_2 are inserted between b and c such that $G_1^3 + G_2^3 = \lambda\, abc$, then λ =
   (a) 1 (b) 2 (c) $\frac{1}{2}$ (d) 3

One or More Than One Option(s) Correct Type

7. If a, b, c are in H.P., then which of the following is/ are true?
   (a) $\frac{a}{b+c-a}, \frac{b}{c+a-b}, \frac{c}{a+b-c}$ are in H.P.
   (b) $\frac{2}{b} = \frac{1}{b-a} + \frac{1}{b-c}$
   (c) $a - \frac{b}{2}, \frac{b}{2}, c - \frac{b}{2}$ are in G.P.
   (d) $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in H.P.

8. Sum to n terms of the series
   $S = 1^2 + 2(2)^2 + 3^2 + 2(4)^2 + 5^2 + 2(6^2) + \ldots$ is
   (a) $1/2n\,(n+1)^2$, when n is even
   (b) $1/2n^2\,(n+1)$, when n is odd
   (c) $1/4n^2\,(n+2)$, when n is odd
   (d) $1/4n(n+2)^2$, when n is even.

9. $\frac{1}{\sqrt{2}+\sqrt{5}} + \frac{1}{\sqrt{5}+\sqrt{8}} + \frac{1}{\sqrt{8}+\sqrt{11}} + \ldots$ upto n terms is equal to

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(a) $\frac{\sqrt{3n+2}-\sqrt{2}}{3}$ (b) $\frac{n}{\sqrt{3n+2}+\sqrt{2}}$

(c) less than n (d) less than $\sqrt{\frac{n}{3}}$

10. Given that $x + y + z = 15$ when a, x, y, z, b are in A.P. and $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=\frac{5}{3}$ when a, x, y, z, b are in H.P. Then

(a) G.M. of a and b is 3

(b) one possible value of $a + 2b$ is 11

(c) A.M. of a and b is 6

(d) greatest value of $a - b$ is 8

11. If d, e, f are in G.P. and the two quadratic equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$ have a common root, then

(a) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in H.P. (b) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(c) $2dbf = aef + cde$ (d) $b^2df = ace^2$

12. The p^{th} term (T_p) of H.P. is $q(p + q)$ and q^{th} term T_q is $p(p + q)$ when $p > 1$, $q > 1$, then

(a) $T_{p+q} = pq$ (b) $T_{pq} = p + q$

(c) $T_{p+q} > T_{pq}$ (d) $T_{pq} > T_{p+q}$

13. If $\frac{1}{a}+\frac{1}{c}=\frac{1}{2b-a}+\frac{1}{2b-c}$, then

(a) a, b, c are in A.P. (b) $a, \frac{b}{2}, c$ are in A.P.

(c) $a, \frac{b}{2}, c$ are in H.P. (d) $a, 2b, c$ are in H.P.

Comprehension Type

Let $a_1, a_2, \ldots$ be an A.P. and $b_1, b_2, \ldots$ be a G.P. The sequence $c_1, c_2, c_3, \ldots$ is such that $c_n = a_n + b_n \; \forall \; n \in N$ and $c_1 = 1$, $c_2 = 4$, $c_3 = 15$, $c_4 = 2$.

14. The common ratio of G.P. is

(a) -2 (b) -3 (c) 2 (d) 3

15. The common difference of A.P. is

(a) 3 (b) 4 (c) 5 (d) 6

Matrix Match Type

16. Let $A_1, A_2, \ldots, A_n$ and $G_1, G_2, \ldots, G_n$ be n A.M.'s and n G.M.'s respectively between x and 1 (where $x \in (0,1)$) such that $\sum_{r=1}^{\infty} x^r = \frac{1}{3}$ and $\sum_{r=1}^{n} A_r = 5$. Then

	Column-I		Column-II
P.	$\sum_{r=1}^{n} r^2$ is divisible by	1.	9
Q.	n is divisible by	2.	8
R.	$\left(\frac{G_n}{G_2}\right)^3$ is divisible by integer	3.	6
S.	Value of $1/x$ is	4.	4

	P	Q	R	S
(a)	3, 2	4, 2	2	2
(b)	2, 4	3	4	1, 4
(c)	3, 4	2, 4	2, 4	4
(d)	3, 4	1, 4	2	4

Numerical Answer Type

17. If the sum of an infinite G.P. is equal to the maximum value of $f(x) = x^3 + 2x - 8$ in the interval $[-1, 4]$ and the sum of first two terms of the G.P. is 8. Then, the common ratio of the G.P. is $\sqrt{\lambda/8}$, where $\lambda =$

18. If nine arithmetic means and nine harmonic means are inserted between 2 and 3 alternatively, then the value of $A+\frac{6}{H}$ (where A is any of the A.M.'s and H the corresponding H.M.) is

19. Consider the 10 numbers $ar, ar^2, ar^3, \ldots ar^{10}$. Their sum is 18 and the sum of their reciprocals is 6. If the product of these 10 numbers is p^q (where p and q are prime), then the value of $p + q$ is

20. Let $a_1, a_2, a_3, \ldots, a_{201}$ are in G.P. with $a_{101} = 25$ and $\sum_{i=1}^{201} a_i = 625$, then the value of $\sum_{i=1}^{201} \frac{1}{a_i}$ equals to

◈◈

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No. of questions attempted .....
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> 90%	EXCELLENT WORK !	You are well prepared to take the challenge of final exam.
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CONCEPT BOOSTER

Class XII

Functions

This column is aimed at Class XII students so that they can prepare for competitive exams such as JEE Main/Advanced, etc. and be also in command of what is being covered in their school as part of NCERT syllabus. The problems here are a happy blend of the straight and the twisted, the simple and the difficult and the easy and the challenging.

*ALOK KUMAR

DEFINITION

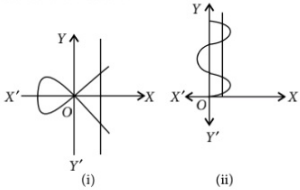
If A and B are two non-empty sets, then a rule f which associates each $x \in A$, to a unique number $y \in B$, is called a function from A to B and we write, $f : A \to B$. We say that 'y' is the image of 'x' under f (or x is the pre-image of y).

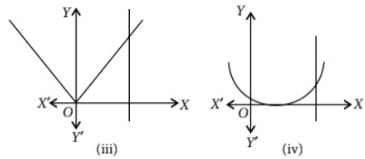
Note : **1.** A mapping $f : X \to Y$ is said to be a function if each element in the set X has its image in set Y. It is also possible that there are few elements in set Y which are not the images of any element in set X.

2. Functions can not be multi-valued *i.e.*, every element in set X should have one and only one image in Y.

Testing for a Function by Vertical Line Test

If it is possible to draw a vertical line which cuts the graph of given curve at more than one point, then the given relation is not a function and when this vertical line means line parallel to Y-axis cuts the curve at only one point then it is a function. Figure (iii) and (iv) represents a function.





NUMBER OF FUNCTIONS

Let X and Y be two finite sets having m and n elements respectively. Then each element of set X can be associated to any one of n elements of set Y. So, total number of functions from set X to set Y is n^m.

DOMAIN, CO-DOMAIN AND RANGE OF FUNCTION

If a function $f : A \to B$ is defined from a set A to set B, then set A is called the domain of function f and set B is called the co-domain of function f. The set of all f-images of the elements of A is called the range of function f.

Methods for finding Domain and Range of Function

- **Domain**

(i) Expression under even root (*i.e.*, square root, fourth root etc.) ≥ 0.

(ii) Denominator $\neq 0$.

(iii) If domain of $y = f(x)$ and $y = g(x)$ are D_1 and D_2 respectively, then

(a) The domain of $f(x) \pm g(x)$ or $f(x) \cdot g(x)$ is $D_1 \cap D_2$.

(b) The domain of $\frac{f(x)}{g(x)}$ is $D_1 \cap D_2 - \{x : g(x) = 0\}$.

(c) Domain of $\left(\sqrt{f(x)}\right) = D_1 \cap \{x : f(x) \geq 0\}$.

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- **Range**

(i) If domain ∈ finite number of points
⇒ Range ∈ set of corresponding values of $f(x)$.

(ii) If domain ∈ R or R – [some finite points], then express x in terms of y and find the values of y for which x exists.

(iii) If domain ∈ a finite interval, then find the least and greatest value for range using monotonicity.

ALGEBRA OF FUNCTIONS

Let $f(x)$ and $g(x)$ be two real and single-valued functions with domains X_f, X_g and ranges Y_f and Y_g respectively. Then, the following operations are defined.

(i) Scalar multiplication of a function :
$(cf)(x) = cf(x)$, where c is a scalar.

(ii) Addition/subtraction of functions :
$(f \pm g)(x) = f(x) \pm g(x)$.

(iii) Multiplication of functions :
$(fg)(x) = (gf)(x) = f(x)g(x)$.

(iv) Division of functions : $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0.$

EQUAL FUNCTIONS

Two functions f and g are said to be equal functions, if and only if

(i) Domain of f = Domain of g

(ii) Co-domain of f = Co-domain of g

(iii) $f(x) = g(x)\ \forall\ x \in$ their common domain

KINDS OF FUNCTION

(i) One-one function (injectivity)

A function $f : A \to B$ is said to be a one-one function or an injection, if different elements of A have different images in B.

Method to check the injectivity of a function

Step I : Take two arbitrary elements x, y (say) in the domain of f.

Step II : Put $f(x) = f(y)$

Step III : If $f(x) = f(y)$ gives $x = y$ only, then $f : A \to B$ is a one-one function (or an injection), otherwise it is not.

Note : 1. If function is given in the form of ordered pairs and if two ordered pairs do not have same second element then function is one-one.

2. If the graph of the function $y = f(x)$ is given and each line parallel to x-axis cuts the given curve at maximum one point, then function is one-one.

3. Any function, which is entirely increasing or decreasing in the whole of a domain is one-one.

- **Number of one-one functions (injections) :** If A and B are finite sets having m and n elements respectively, then number of one-one functions from A to $B = \begin{cases} {}^nP_m, & \text{if } n \geq m \\ 0\ , & \text{if } n < m \end{cases}$

(ii) Many-one function

A function $f : A \to B$ is said to be a many-one function if two or more elements of set A have the same image in B.

Thus, $f : A \to B$ is a many-one function if there exist $x, y \in A$ such that $x \neq y$ but $f(x) = f(y)$

Note : 1. If function is given in the form of set of ordered pairs and the second element of atleast two ordered pairs are same, then function is many-one.

2. If the graph of $y = f(x)$ is given and the line parallel to x-axis cuts the curve at more than one point, then function is many-one.

3. If the domain of the function is in one quadrant, then the trigonometric functions are always one-one.

4. If trigonometric function changes its sign in two consecutive quadrants, then it is one-one but if it does not change the sign, then it is many-one.

5. In three consecutive quadrants, trigonometric functions are always many-one.

6. Any continuous function $f(x)$, which has atleast one local maximum or local minimum is many-one.

(iii) Onto function (Surjectivity)

A function $f : A \to B$ is onto if each element of B has its pre-image in A.

In other words,

range of f = co-domain of f.

- **Number of onto functions (surjections) :** If A and B are two sets having m and n elements respectively such that $1 \leq n \leq m$, then number of onto functions from A to B is $\sum_{r=1}^{n} (-1)^{n-r}\ {}^nC_r\, r^m$.

(iv) Into function

A function $f : A \to B$ is an into function if there exists an element in B having no pre-image in A. In other words, $f : A \to B$ is an into function if it is not an onto function.

Method to find onto or into function

(a) If range = co-domain, then $f(x)$ is onto and if range is a proper subset of the co-domain, then $f(x)$ is into.

(b) Solve $f(x) = y$ by taking x as a function of y *i.e.*, $g(y)$ (say). Now, if $g(y)$ is defined for each $y \in$ co-domain and $g(y) \in$ domain for $y \in$ co-domain, then $f(x)$ is onto.

Note : 1. An into function can be made onto by redefining the co-domain as the range of the original function.

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2. Any polynomial function $f : R \to R$ is onto if degree of f is odd and into if degree of f is even.

(v) One-one Onto function (Bijection)

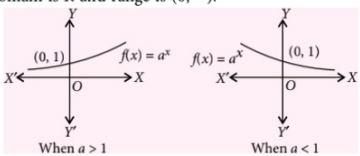
A function $f : A \to B$ is a bijection if it is one-one as well as onto. In other words, a function $f : A \to B$ is a bijection, if for $f(x) = f(y) \Rightarrow x = y$ for all $x, y \in A$ and for all $y \in B$, there exists $x \in A$ such that $f(x) = y$.

- **Number of one-one onto functions (bijections) :** If A and B are finite sets and $f : A \to B$ is a bijection, then A and B have the same number of elements. If A has n elements, then number of bijections from A to B is $n!$

SOME SPECIAL FUNCTIONS

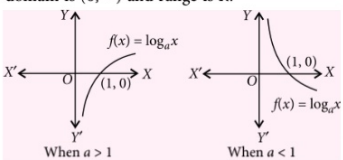
(i) Exponential function

Let $a \neq 1$ be a positive real number. Then $f : R \to (0, \infty)$ defined by $f(x) = a^x$ is called exponential function. Its domain is R and range is $(0, \infty)$.



(ii) Logarithmic function

Let $a \neq 1$ be a positive real number. Then $f : (0, \infty) \to R$ defined by $f(x) = \log_a x$ is called logarithmic function. Its domain is $(0, \infty)$ and range is R.

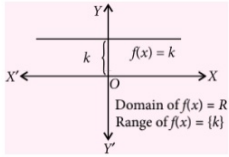


(iii) Explicit and implicit functions

A function is said to be explicit, if it can be expressed directly in terms of the independent variable. If the function can not be expressed directly in terms of the independent variable or variables, then the function is said to be implicit *e.g.* $y = \sin^{-1}x + \log x$ is an explicit function while $x^2 + y^2 = xy$ and $x^3y^2 = (a - x)^2(b - y)^2$ are implicit functions.

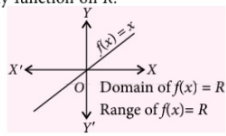
(iv) Constant function

Let k be a fixed real number. Then a function $f(x)$ given by $f(x) = k$ for all $x \in R$ is called a constant function.



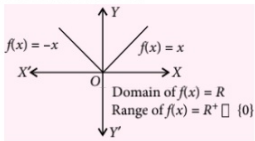
(v) Identity function

The function defined by $f(x) = x$ for all $x \in R$ is called the identity function on R.



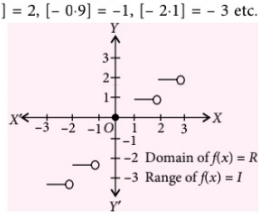
(vi) Modulus function

The function defined by $f(x) = |x| = \begin{cases} x, & \text{when } x \geq 0 \\ -x, & \text{when } x < 0 \end{cases}$ is called the modulus function.



(vii) Greatest integer function

The function f defined by $f(x) = [x]$ for all $x \in R$, is called the greatest integer function. Here $[x]$ denotes the greatest integer less than or equal to x for *e.g.* $[1{\cdot}1] = 1$, $[2{\cdot}2] = 2$, $[-0{\cdot}9] = -1$, $[-2{\cdot}1] = -3$ etc.



(viii) Signum function

The function defined by

$$f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases} \quad \text{or} \quad f(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

is called the signum function.

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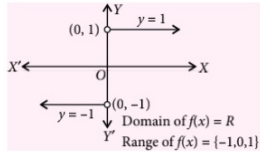
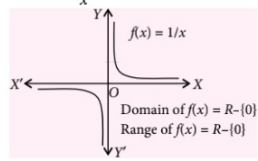


(ix) Reciprocal function

The function that associates each non-zero real number x to the reciprocal $\frac{1}{x}$ is called the reciprocal function.



EVEN AND ODD FUNCTION

(i) Even function

If we put $(-x)$ in place of x in the given function and if $f(-x) = f(x)$, $\forall\ x \in$ domain, then $f(x)$ is called even function.

e.g. $f(x) = e^x + e^{-x}$, $f(x) = x^2$, $f(x) = x\sin x$, $f(x) = \cos x$ etc. are all even functions.

(ii) Odd function

If we put $(-x)$ in place of x in the given function and if $f(-x) = -f(x)$ $\forall\ x \in$ domain, then $f(x)$ is called odd function.

e.g., $f(x) = e^x - e^{-x}$, $f(x) = \sin x$, $f(x) = x^3$, $f(x) = x\cos x$, $f(x) = x^2 \sin x$ etc. are all odd functions.

Properties of Even and Odd Functions

- The graph of an even function is always symmetric with respect to y-axis.
- The graph of an odd function is always symmetric with respect to origin.
- The sum and difference of two even functions is an even function.
- The product of two even functions is an even function.
- The sum and difference of two odd functions is an odd function.
- The product of two odd functions is an even function.
- The product of an even and an odd function is an odd function
- It is not essential that every function is even or odd. It is possible to have some functions which are neither even nor odd function.
  e.g. $f(x) = x^2 + x^3$ and $f(x) = e^x$.
- The sum of an even and an odd function is neither even nor odd function.

PERIODIC FUNCTION

A function is said to be periodic function, if its each value is repeated after a definite interval. So, a function $f(x)$ will be periodic if there exists a positive real number T such that $f(x + T) = f(x)$, $\forall\ x \in$ domain. Here, the least positive value of T is called the period of the function. Clearly $f(x) = f(x + T) = f(x + 2T) = f(x + 3T) = ...$

e.g. $\sin x$, $\cos x$, $\tan x$ are periodic functions with period 2π, 2π and π respectively.

Some standard results on periodic functions

	Functions	Periods
1.	$\sin^n x$, $\cos^n x$, $\sec^n x$, $\operatorname{cosec}^n x$	π, if n is even 2π, if n is odd or fraction
2.	$\tan^n x$, $\cot^n x$	π, n is even or odd
3.	$\lvert\sin x\rvert$, $\lvert\cos x\rvert$, $\lvert\tan x\rvert$, $\lvert\cot x\rvert$, $\lvert\sec x\rvert$, $\lvert\operatorname{cosec} x\rvert$	π
4.	$x - [x]$	1

COMPOSITE FUNCTION

If $f : A \to B$ and $g : B \to C$ are two functions, then the composite function of f and g, given by $gof : A \to C$ will be defined as $gof(x) = g[f(x)]$ $\forall\ x \in A$.

Properties of Composition of Function

- f is even, g is even $\Rightarrow$ fog is even function.
- f is odd, g is odd $\Rightarrow$ fog is odd function.
- f is even, g is odd $\Rightarrow$ fog is even function.
- f is odd, g is even $\Rightarrow$ fog is even function.
- Composition of functions is not commutative, *i.e.*, $fog \neq gof$.
- Composition of functions is associative *i.e.*, $(fog)oh = fo(goh)$.
- $gof(x)$ is simply the g-image of $f(x)$, where $f(x)$ is f-image of elements $x \in A$.
- Function gof will exist only when range of f is the subset of domain of g.
- fog and gof may not be always defined. fog does not exist if range of g is not a subset of domain of f.
- If both f and g are one-one, then fog and gof are also one-one.
- If both f and g are onto, then gof is onto.

INVERSE OF A FUNCTION

If $f : A \to B$ be a one-one onto (bijection) function, then the mapping $f^{-1} : B \to A$ which associates each

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element $b \in B$ with element $a \in A$ such that $f(a) = b$ is called the inverse of the function $f: A \to B$ and denoted by $f^{-1}: B \to A$ as $f^{-1}(b) = a \Rightarrow f(a) = b$

Important Points for Inverse Function

- For the existence of inverse function, it should be one-one and onto.
- Inverse of a bijection is also a bijection.
- Inverse of a bijection is unique.
- $(f^{-1})^{-1} = f$
- If $f: A \to B$ and $g: B \to C$ are two bijections, then $gof: A \to C$ is a bijection and $(gof)^{-1} = (f^{-1}og^{-1})$
- $fog \neq gof$ but if $fog = gof$, then either $f^{-1} = g$ or $g^{-1} = f$. Also, $(fog)(x) = (gof)(x) = (x)$.
- If $f: A \to B$ is a bijection then $f^{-1}: B \to A$ is an inverse function of f. $f^{-1}of = I_A$ and $fof^{-1} = I_B$. Here I_A, is an identity function on set A and I_B is an identity function on set B.

PROBLEMS

Single Correct Answer Type

1. If $f(x) = \frac{1}{\sqrt{x+2\sqrt{2x-4}}} + \frac{1}{\sqrt{x-2\sqrt{2x-4}}}$ for $x > 2$, then $f(11) =$

(a) 7/6 (b) 5/6 (c) 6/7 (d) 5/7

2. If for two functions g and f, gof is both injective and surjective, then which of the following is true?

(a) g and f should be injective and surjective
(b) g should be injective and surjective
(c) f should be injective and surjective
(d) none of these

3. The function $f: R \to R$, defined by $f(x) = x^2 + x$ is

(a) one-one onto (b) one-one into
(c) many-one onto (d) many-one into

4. If $f(x, y) = \frac{x}{y}$ where $(x, y) \in R$ and $x, y \neq 0$, then this function is a/an

(a) surjective (b) bijective
(c) one-one (d) none of these

5. If $f(x)$ is periodic function with period T then the function $f(ax + b)$ where $a > 0$, is periodic with period

(a) T/b (b) aT (c) bT (d) T/a

6. Domain of $f(x) = \log|\log x|$ is

(a) $(0, \infty)$ (b) $(1, \infty)$
(c) $(0, 1) \cup (1, \infty)$ (d) $(-\infty, 1)$

7. The domain of the function $f(x) = \sin^{-1}[\log_2(x/2)]$ is

(a) $[1, 4]$ (b) $[-4, 1]$
(c) $[-1, 4]$ (d) none of these

8. The domain of the function $f(x) = \frac{\log_2(x+3)}{x^2+3x+2}$ is

(a) $R - \{-1, -2\}$ (b) $(-2, \infty)$
(c) $R - \{-1, -2, -3\}$ (d) $(-3, \infty) - \{-1, -2\}$

9. The domain of the function $f(x) = \sqrt{\log \frac{1}{|\sin x|}}$ is

(a) $R - \{2n\pi, n \in I\}$ (b) $R - \{n\pi, n \in I\}$
(c) $R - \{-\pi, \pi\}$ (d) $(-\infty, \infty)$

10. The domain of the function $f(x) = \log(\sqrt{x-4} + \sqrt{6-x})$ is

(a) $[4, \infty)$ (b) $(-\infty, 6]$
(c) $[4, 6]$ (d) none of these

11. The domain of the derivative of the function

$$f(x) = \begin{cases} \tan^{-1} x & , |x| \leq 1 \\ \frac{1}{2}(|x| - 1) & , |x| > 1 \end{cases}$$ is

(a) $R - \{0\}$ (b) $R - \{1\}$
(c) $R - \{-1\}$ (d) $R - \{-1, 1\}$

12. The domain of the function $f(x) = \log_{3+x}(x^2 - 1)$ is

(a) $(-3, -1) \cup (1, \infty)$ (b) $[-3, -1] \cup [1, \infty)$
(c) $(-3, -2) \cup (-2, -1) \cup (1, \infty)$
(d) $[-3, -2) \cup (-2, -1) \cup [1, \infty)$

13. If n is an integer, then the domain of the function $f(x) = \sqrt{\sin 2x}$ is

(a) $\left[n\pi - \frac{\pi}{2}, n\pi\right]$ (b) $\left[n\pi, n\pi + \frac{\pi}{2}\right]$
(c) $[(2n-1)\pi, 2n\pi]$ (d) $[2n\pi, (2n+1)\pi]$

14. Domain of the function $\sqrt{\log\{(5x - x^2)/6\}}$ is

(a) $(2, 3)$ (b) $[2, 3]$ (c) $[1, 2]$ (d) $[1, 3]$

15. The domain of the function $y = \frac{1}{\sqrt{|x| - x}}$ is

(a) $(-\infty, 0)$ (b) $(-\infty, 0]$ (c) $(-\infty, -1)$ (d) $(-\infty, \infty)$

16. The function $f: R \to R$, $f(x) = x^2$ $\forall x \in R$ is

(a) injective but not surjective
(b) surjective but not injective
(c) injective as well as surjective
(d) neither injective nor surjective

17. If $f(x) = \frac{x}{1+x}$, then $f^{-1}(x)$ is equal to

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(a) $\frac{(1+x)}{x}$ (b) $\frac{1}{(1+x)}$ (c) $\frac{(1+x)}{(1-x)}$ (d) $\frac{x}{(1-x)}$

18. The inverse of the function $\frac{10^x-10^{-x}}{10^x+10^{-x}}$ is

(a) $\frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right)$ (b) $\frac{1}{2}\log_{10}\left(\frac{1-x}{1+x}\right)$

(c) $\frac{1}{4}\log_{10}\left(\frac{2x}{2-x}\right)$ (d) none of these

19. If $f(x)=\frac{x}{\sqrt{1+x^2}}$, then $(fofof)(x)$ =

(a) $\frac{3x}{\sqrt{1+x^2}}$ (b) $\frac{x}{\sqrt{1+3x^2}}$

(c) $\frac{x}{\sqrt{1+x^2}}$ (d) none of these

20. If $g(x) = x^2 + x - 2$ and $\frac{1}{2}(gof)(x)=2x^2-5x+2$, then $f(x)$ is

(a) $2x - 3$ (b) $2x + 3$

(c) $2x^2 + 3x + 1$ (d) $2x^2 - 3x - 1$

21. If $f: R \to R$ and $g: R \to R$ defined by $f(x) = (x + 1)^2$, $g(x) = x^2 + 1$, then $(fog)(-3)$ equals

(a) 121 (b) 112

(c) 211 (d) none of these

22. Let $f(x) = ax + b$ and $g(x) = cx + d$, $a \neq 0$, $c \neq 0$. Assume $a = 1$, $b = 2$. If $(fog)(x) = (gof)(x)$ for all x, then what can you say about c and d ?

(a) c and d both arbitrary (b) $c = 1$, d arbitrary

(c) c arbitrary, $d = 1$ (d) none of these

23. Let $g(x) = 1 + x - [x]$ and $f(x)=\begin{cases}-1, & x<0\\ 0, & x=0,\\ 1, & x>0\end{cases}$ then for all x, $f(g(x))$ is equal to

(a) x (b) 1 (c) $f(x)$ (d) $g(x)$

24. If $f(x)=\frac{\alpha x}{x+1}, x \neq -1$. Then, value of α for which $f(f(x)) = x$ is

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d) -1

25. Let $f : (-1, 1) \to B$ be a function defined by $f(x)=\tan^{-1}\frac{2x}{1-x^2}$, then f is both one-one and onto when B is the interval

(a) $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ (b) $\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

(c) $\left(0,\frac{\pi}{2}\right)$ (d) $\left[0,\frac{\pi}{2}\right)$

26. If $f(x) = 2x^6 + 3x^4 + 4x^2$, then $f'(x)$ is

(a) an even function (b) an odd function

(c) neither even nor odd (d) can't be determined

27. If a real valued function $f(x)$ satisfies the functional equation $f(x - y) = f(x) f(y) - f(a - x) f(a + y)$, where a is a given constant and $f(0) = 1$ then, $f(2a - x)$ is equal to

(a) $f(a) + f(a - x)$ (b) $f(-x)$

(c) $-f(x)$ (d) $f(x)$

28. If $g : [-2, 2] \to R$ where $g(x)=x^3+\tan x+\left[\frac{x^2+1}{P}\right]$ is an odd function where $[\cdot]$ denotes greatest integer function, then the value of parameter P is

(a) $-5 < P < 5$ (b) $P < 5$

(c) $P > 5$ (d) none of these

29. If $f(x)=\begin{cases}x, & \text{when } x \text{ is rational}\\ 0, & \text{when } x \text{ is irrational}\end{cases}$

and $g(x)=\begin{cases}0, & \text{when } x \text{ is rational}\\ x, & \text{when } x \text{ is irrational}\end{cases}$, then $(f - g)$ is

(a) one-one onto (b) one-one not onto

(c) not one-one but onto

(d) neither one-one nor onto

30. Range of the function $f : R \to R$ given by

$f(x)=\frac{x^2}{x^2+1}$ is

(a) $(-1, 0)$ (b) $(-1, 1)$ (c) $[0, 1)$ (d) $(1, 1)$

31. The function f satisfies the functional equation $3f(x)+2f\left(\frac{x+59}{x-1}\right)=10x+30$ for all real $x \neq 1$. The value of $f(7)$ is

(a) 8 (b) 4 (c) -8 (d) 11

Multiple Correct Answer Type

32. If $f(x_1)-f(x_2)=f\left(\frac{x_1-x_2}{1-x_1x_2}\right)$ for $x_1, x_2 \in [-1, 1]$, then $f(x)$ is

(a) $\log\frac{(1-x)}{(1+x)}$ (b) $\tan^{-1}\frac{(1-x)}{(1+x)}$

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(c) $\log\frac{(1+x)}{(1-x)}$ (d) $\tan^{-1}\frac{(1+x)}{(1-x)}$

33. Let $f(x)=\frac{\sin x}{\sqrt{1+\tan^2 x}}+\frac{\cos x}{\sqrt{1+\cot^2 x}}$, then

(a) fundamental period of $f(x)$ is 2π
(b) fundamental period of $f(x)$ is π
(c) domain of $f(x)$ is R
(d) range of $f(x)$ is $[-1, 1]$

34. Let $f : R \to R$ be a function defined by $f(x+1)=\frac{f(x)-5}{f(x)-3}, \forall x \in R$. Then which of the following statement(s) is/are true?

(a) $f(x+2)=f(x)$ (b) $f(x+4)=f(x)$
(c) $f(x+6)=f(x)$ (d) $f(x+8)=f(x)$

35. Let $R = \{(x, y) : x, y \in R, x^2 + y^2 \le 25\}$ and $R'=\left\{(x, y): x, y \in R, y \ge \frac{4}{9}x^2\right\}$. Then,

(a) domain of $R \cap R' = [-3, 3]$
(b) range of $R \cap R' = [0, 4]$
(c) range of $R \cap R' = [0, 5]$
(d) none of these

36. Let f be a function defined by $f(x)=\frac{x-5}{x-3}$, $x \ne 3, 2$; $f^k(x)$ denote the composition of f with itself taken k times *i.e.*, $f^3(x) = f(f(f(x)))$. Then,

(a) $f^{2012}(2009) = 2009$ (b) $f^{2009}(2010)=\frac{2006}{2007}$

(c) $f^{2009}(2011)=\frac{1003}{1004}$ (d) $f^{2012}(2012) = 2012$

Comprehension Type

Paragraph for Q. No. 37-38

A function $f(x)$ is said to be even iff $f(-x) = f(x)$ and odd iff $f(-x) = -f(x)$. Also $f(x)$ is said to be periodic iff $f(x + T) = f(x)$, $T > 0$ and the least positive value T is called its period.

37. $f(x)$ is an even polynomial function. Then $\sin(f(x) - 3x)$ is

(a) an even function
(b) an odd function
(c) neither even nor odd
(d) periodic with period $\pi/2$

38. If $f(x)$ and $g(x)$ be two functions with all real numbers as their domains, then $h(x)=[f(x)+f(-x)][g(x)-g(-x)]$ is

(a) an even function when f is even and g is odd
(b) an even function when both f and g are odd
(c) always an odd function
(d) always an even function

Paragraph for Q. No. 39-40

Let us consider the two non empty sets X and Y such that $X = \{1, 2, 3, 4, 5, 6, 7\}$ and $Y = \{1, 2, 3, 4, 5, 6\}$.

39. The number of many one functions from X to Y, is

(a) 6^7 (b) 7^6 (c) $7^6 - 6^7$ (d) $6^7 - 6!$

40. The number of functions from X to Y and such that $f(x_i) \ne x_i \ \forall\ x_i \in \{1, 2, 3, \ldots, 7\}$, is

(a) $6(5)^6$ (b) $5(6)^5$
(c) $5(7)^6$ (d) none of these

Matrix-Match Type

41. Match the following:

	Column- I		Column- II
A.	If f is a function such that $f(0) = 2$, $f(1) = 3$ and $f(x + 2) = 2f(x) - f(x + 1)$, then $f(5)$ is equal to	(p)	4
B.	If $f(x)=\begin{cases}x^2, & \text{for } x \ge 0\\ x, & \text{for } x<0\end{cases}$, then $f(\sqrt{13})=$	(q)	3
C.	If $f(x) + 2f(1 - x) = x^2 + 2$ for all $x \in R$, then $f(5)$ is	(r)	12
D.	If $f(x)=\frac{4^x}{4^x+2}$ for all $x \in R$, then $\sum_{k=1}^{6} f\left(\frac{k}{7}\right)$ equals	(s)	2
		(t)	13

42. Match the following:

	Column-I		Column-II
A.	Let $f : R \to R$ satisfies $f(x + y) + f(x - y) = 2f(x) f(y)\ \forall\ x, y \in R$ and $f(0) \ne 0$, then $f(x)$ is	(p)	even
B.	Let $f : R \to R$ is defined by $f(x)=\frac{e^{\lvert x\rvert}-e^{-x}}{e^x+e^{-x}}$, then $f(x)$ is	(q)	odd

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C.	Let $f: R \to R$ be a polynomial function satisfying $f(x)f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$ and $f(3)=28$, then $f(x)$ is	(r)	into
D.	Let $f: R \to R$ is defined by $f(x) = 2x + \sin x$, then $f(x)$ is	(s)	many-one
		(t)	bijective

Numerical Answer Type

43. If n is a natural number and $1 \le n \le 100$, then the number of solutions of $\left[\frac{n}{2}\right]+\left[\frac{n}{3}\right]+\left[\frac{n}{5}\right]=\frac{n}{2}+\frac{n}{3}+\frac{n}{5}$ (where [.] denotes the greatest integer function) is _____.

44. The number of solutions of the equation $x^2 + [x] - 4x + 3 = 0$ (where [.] denotes the greatest integer function) is_____.

45. If the function $f(x)=\begin{cases}x+1, & x\le 1\\ 2x+1, & 1<x\le 2\end{cases}$ and $g(x)=\begin{cases}x^2, & -1\le x\le 2\\ x+2, & 2\le x\le 3\end{cases}$ then the number of roots of the equation $f(g(x)) = 2$ is _____.

SOLUTIONS

1. (c) : $f(x)=\frac{1}{\sqrt{x+2\sqrt{2x-4}}}+\frac{1}{\sqrt{x-2\sqrt{2x-4}}}$

$\therefore\ f(11)=\frac{1}{\sqrt{11+2\sqrt{18}}}+\frac{1}{\sqrt{11-2\sqrt{18}}}$

$=\frac{1}{3+\sqrt{2}}+\frac{1}{3-\sqrt{2}}=\frac{3-\sqrt{2}}{7}+\frac{3+\sqrt{2}}{7}=\frac{6}{7}$

2. (a) : As gof is bijective, then g and f are also bijective.

3. (d) : $\because\ f(0) = f(-1) = 0 \Rightarrow f(x)$ is many one. Also, there is no pre-image of -1. Hence, $f(x)$ is into function.

4. (a) : $\because\ f(\alpha x, \alpha y)=\frac{\alpha x}{\alpha y}=\frac{x}{y}(\alpha\neq 0)$.

Hence, $f(x, y)$ is not an injective function.

5. (d) : $f\left(a\left(x+\frac{T}{a}\right)+b\right)=f(ax+b+T)=f(ax+b)$.

6. (c) : $f(x) = \log|\log x|$, $f(x)$ is defined if $|\log x| > 0$ and $x > 0$.

i.e., $x > 0$ and $x \neq 1$ $\quad(\because\ |\log x| > 0$, if $x \neq 1)$

$\therefore\ x \in (0, 1) \cup (1, \infty)$

7. (a) : We have, $f(x) = \sin^{-1}[\log_2(x/2)]$.

Domain of $\sin^{-1}x$ is $[-1, 1]$

$\Rightarrow\ -1\le\log_2(x/2)\le 1\ \Rightarrow\ \frac{1}{2}\le\frac{x}{2}\le 2\ \Rightarrow\ 1\le x\le 4$

$\therefore\ x \in [1, 4]$.

8. (d) : Here, $x + 3 > 0$ and $x^2 + 3x + 2 \neq 0$

$\therefore\ x > -3$ and $(x + 1)(x + 2) \neq 0$, *i.e.*, $x \neq -1, -2$

$\therefore$ Domain $= (-3, \infty) - \{-1, -2\}$.

9. (b) : We have, $f(x)=\sqrt{\log\frac{1}{|\sin x|}}$

$\Rightarrow\ \sin x \neq 0\ \Rightarrow\ x \neq n\pi, n \in I$

$\therefore$ Domain of $f(x) = R - \{n\pi, n \in I\}$.

10. (c) : $f(x)=\log(\sqrt{x-4}+\sqrt{6-x})$

$\Rightarrow\ \sqrt{x-4}+\sqrt{6-x}>0$

$\therefore$ Domain of $f(x) = [4, 6]$.

11. (c) : We have, $f(x)=\begin{cases}\frac{1}{2}(-x-1), & x<-1\\ \tan^{-1}x, & -1\le x\le 1;\\ \frac{1}{2}(x-1), & x>1\end{cases}$

$\therefore\ f'(x)=\begin{cases}-\frac{1}{2}, & x<-1\\ \frac{1}{1+x^2}, & -1\le x\le 1\\ \frac{1}{2}, & x>1\end{cases}$

Now, $L[f'(-1)]=-\frac{1}{2}$ and $R[f'(-1)]=\frac{1}{1+(-1)^2}=\frac{1}{2}$

Also, $L[f'(1)]=\frac{1}{1+(1)^2}=\frac{1}{2}$; $R[f'(1)]=\frac{1}{2}$

$\because\ f'(-1)$ does not exist.

$\therefore$ Domain of $f(x) = R - \{-1\}$.

12. (c) : We have, $f(x) = \log_{3+x}(x^2 - 1)$

$\therefore\ x^2 > 1 \Rightarrow x < -1$ or $x > 1$ and $3 + x > 0$

$\Rightarrow\ x > -3$. Also, $3 + x \neq 1 \Rightarrow x \neq -2$

$\therefore\ D_f = (-3, -2) \cup (-2, -1) \cup (1, \infty)$.

13. (b) : $\because \sin 2x$ can't be negative.

$\therefore$ Domain of function $\sqrt{\sin 2x}$ is $\left[n\pi, n\pi+\frac{\pi}{2}\right]$.

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14. (b) : $\log\left\{\frac{5x-x^2}{6}\right\}\geq 0 \Rightarrow \frac{5x-x^2}{6}\geq 1$

$\Rightarrow \quad x^2-5x+6\leq 0 \Rightarrow 2\leq x\leq 3$

15. (a) : We have, $y=\frac{1}{\sqrt{|x|-x}}$

$\Rightarrow \quad |x|-x>0$ or $|x|>x$

But $|x|=x$ for all positive x and $|x|>x$ for all negative x.

So, domain will be $(-\infty, 0)$.

16. (d) : The mapping $f(x)=x^2$ is many-one into.

17. (d) : Let $y=f(x)=\frac{x}{1+x} \Rightarrow x=f^{-1}(y)$

$\therefore \quad y=\frac{x}{1+x} \Rightarrow y+yx=x \Rightarrow x=\frac{y}{1-y}$

$\Rightarrow \quad f^{-1}(y)=\frac{y}{1-y} \Rightarrow f^{-1}(x)=\frac{x}{1-x}.$

18. (a) : Let $y=f(x)=\frac{10^x-10^{-x}}{10^x+10^{-x}}$

$\Rightarrow \quad x=\frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right)$

$\Rightarrow \quad f^{-1}(y)=\frac{1}{2}\log_{10}\left(\frac{1+y}{1-y}\right)$ $(\because y=f(x) \Rightarrow x=f^{-1}(y))$

$\therefore \quad f^{-1}(x)=\frac{1}{2}\log_{10}\left(\frac{1+x}{1-x}\right).$

19. (b) : $(fofof)(x)=(fof)(f(x))=(fof)\left(\frac{x}{\sqrt{1+x^2}}\right)$

$$=f\left[\frac{\left(\frac{x}{\sqrt{1+x^2}}\right)}{\sqrt{1+\frac{x^2}{1+x^2}}}\right]=f\left(\frac{x\sqrt{1+x^2}}{\sqrt{1+x^2}\sqrt{1+2x^2}}\right)$$

$$=f\left(\frac{x}{\sqrt{1+2x^2}}\right)=\frac{\frac{x}{\sqrt{1+2x^2}}}{\sqrt{\left[1+\frac{x^2}{1+2x^2}\right]}}=\frac{x}{\sqrt{1+3x^2}}.$$

20. (a) : Given, $\frac{1}{2}(gof)(x)=2x^2-5x+2$

or $\frac{1}{2}g[f(x)]=2x^2-5x+2$

$\therefore \quad [\{f(x)\}^2+\{f(x)\}-2]=2[2x^2-5x+2]$

$\Rightarrow \quad [f(x)]^2+f(x)-(4x^2-10x+6)=0$

$$\therefore \quad f(x)=\frac{-1\pm\sqrt{1+4(4x^2-10x+6)}}{2}$$

$$=\frac{-1\pm\sqrt{(16x^2-40x+25)}}{2}=\frac{-1\pm(4x-5)}{2}=2x-3, 2-2x$$

21. (a) : $f(x)=(1+x)^2$ and $g(x)=x^2+1$

Also, $fog(-3)=f[g(-3)]=f[9+1]=f[10]=[11]^2=121.$

22. (b) : $(fog)(x)=f(g(x))=a(cx+d)+b$

and $(gof)(x)=g(f(x))=c(ax+b)+d$

Given that, $(fog)(x)=(gof)(x)$

At $a=1$, $b=2$, we have

$cx+d+2=cx+2c+d$

$\Rightarrow \quad c=1$ and d is arbitrary.

23. (b) : Here $g(x)=1+n-n=1, x=n\in Z$

$g(x)=1+n+k-n=1+k, x=n+k$

(where $n\in Z$, $0<k<1$)

Clearly, $g(x)>0$ for all x. So, $f(g(x))=1$ for all x.

24. (d) : $f(f(x))=\frac{\alpha f(x)}{f(x)+1}=\frac{\alpha\left(\frac{\alpha x}{x+1}\right)}{\left(\frac{\alpha x}{x+1}+1\right)}=\frac{\alpha^2\cdot x}{\alpha x+x+1}$

Now, $x=\frac{\alpha^2\cdot x}{(\alpha+1)x+1} \Rightarrow x[(\alpha+1)x+1-\alpha^2)]=0$

or $(\alpha+1)x^2+(1-\alpha^2)x=0.$

This should hold for all x.

$\Rightarrow \quad \alpha+1=0, 1-\alpha^2=0 \Rightarrow \alpha=-1.$

25. (b) : For $-1<x<1, \tan^{-1}\frac{2x}{1-x^2}=2\tan^{-1}x$

Range of $f(x)=\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

$\therefore \quad$ Co-domain of function $=B=\left(-\frac{\pi}{2},\frac{\pi}{2}\right).$

26. (b) : We have, $f(x)=2x^6+3x^4+4x^2$

$f(-x)=2(-x)^6+3(-x)^4+4(-x)^2=f(x)$

$\therefore \quad f(x)$ is an even function and derivative of an even function is always odd.

27. (c) : $f(a-(x-a))=f(a)f(x-a)-f(0)f(x) \quad ...(i)$

Put $x=0$, $y=0$; $f(0)=(f(0))^2-[f(a)]^2 \Rightarrow f(a)=0$

From (i), $f(2a-x)=-f(x).$ $[\because f(0)=1]$

28. (c) : Given, $g(x)=x^3+\tan x+\left[\frac{x^2+1}{P}\right]$

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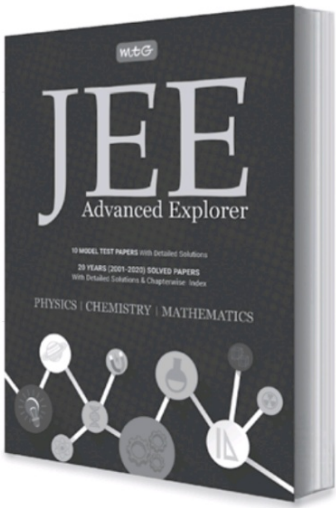

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$\therefore \quad g(-x) = -x^3 - \tan x + \left[\frac{x^2+1}{P}\right]$

Now, $g(x) + g(-x) = 0 \quad [\because \ g(x)$ is an odd function$]$

$\therefore \quad x^3 + \tan x + \left[\frac{x^2+1}{P}\right] - x^3 - \tan x + \left[\frac{x^2+1}{P}\right] = 0$

$\Rightarrow \quad 2\left[\frac{x^2+1}{P}\right] = 0 \Rightarrow 0 \le \frac{x^2+1}{P} < 1$

Since, $x \in [-2, 2] \Rightarrow \quad 0 \le \frac{5}{P} < 1 \Rightarrow P > 5.$

29. (a) : $(f-g)(x) = \begin{cases} x, & x \in Q \\ -x, & x \notin Q \end{cases}$

30. (c) : Let $y = \frac{x^2}{x^2+1}$

$\Rightarrow \quad (y-1)x^2 + 0x + y = 0, \ y \neq 1$

For real value of x, $D \ge 0$

$\Rightarrow -4y(y-1) \ge 0$

$\Rightarrow \quad y(y-1) \le 0$

$\Rightarrow \quad y \le [0, 1) \quad \left[\because \ 0 \le \frac{x^2}{x^2+1} < 1\right]$

31. (b) : We have, $3f(x) + 2f\left(\frac{x+59}{x-1}\right) = 10x + 30$

For $x = 7$, $3f(7) + 2f(11) = 70 + 30 = 100$

For $x = 11$, $3f(11) + 2f(7) = 140$

$\therefore \quad \frac{f(7)}{-20} = \frac{f(11)}{-220} = \frac{-1}{9-4} \Rightarrow f(7) = 4.$

32. (a,b,c) : When $x_1 = -1$ and $x_2 = 1$, then

$f(-1) - f(1) = f\left[\frac{-1-1}{1+1(1)}\right] = f(-1) \Rightarrow f(1) = 0,$

which is satisfied when $f(x) = \tan^{-1}\left(\frac{1-x}{1+x}\right)$

When $x_1 = x_2 = 0$, then

$f(0) - f(0) = f\left[\frac{0-0}{1-0}\right] = f(0) \Rightarrow f(0) = 0 \qquad ...(i)$

Also, when $x_1 = -1$ and $x_2 = 0$ then

$f(-1) - f(0) = f\left(\frac{-1-0}{1-0}\right) = f(-1) \Rightarrow f(0) = 0 \qquad ...(ii)$

Both (i) and (ii) are satisfied when

$f(x) = \log\left(\frac{1-x}{1+x}\right)$ and $f(x) = \log\left(\frac{1+x}{1-x}\right)$

33. (a,d) : $f(x) = \begin{cases} \sin 2x, & 0 < x < \frac{\pi}{2} \\ 0, & \frac{\pi}{2} < x < \pi \\ -\sin 2x, & \pi < x < \frac{3\pi}{2} \\ 0, & \frac{3\pi}{2} < x < 2\pi \end{cases}$

So, $f(x)$ is periodic with fundamental period 2π and range $[-1, 1]$.

34. (b, d) : Given, $f(x+1) = \frac{f(x)-5}{f(x)-3} \qquad ...(i)$

$\Rightarrow \quad f(x) = \frac{3f(x+1)-5}{f(x+1)-1}$

Replace 'x' by $x - 1$, $f(x-1) = \frac{3f(x)-5}{f(x)-1} \qquad ...(ii)$

Replace 'x' by $x + 1$ in (i)

$f(x+2) = \frac{f(x+1)-5}{f(x+1)-3} = \frac{2f(x)-5}{f(x)-2} \qquad ...(iii)$

Similarly, $f(x-2) = \frac{2f(x)-5}{f(x)-2}$

$\therefore \quad f(x+2) = f(x-2) \Rightarrow f(x+4) = f(x)$

and $f(x+8) = f(x+4) = f(x)$

35. (a, c) : We have, $x^2 + y^2 \le 25$

And $9y \ge 4x^2 \Rightarrow x^2 \le \frac{9}{4}y$

Consider $x^2 = \frac{9y}{4}$

Now, $\frac{9y}{4} + y^2 - 25 = 0$

$\Rightarrow \quad 4y^2 + 9y - 100 = 0$

$\Rightarrow \quad (y-4)\left(y + \frac{50}{8}\right) = 0$

$\Rightarrow \quad y = 4$

Domain of $R \cap R' = [-3, 3]$

Range of $R \cap R' = [0, 5]$

36. (a, c, d) : $f(x) = \frac{x-5}{x-3} \Rightarrow f^2(x) = \frac{2x-5}{x-2} = g(x)$

Then, $g^{2n}(x) = x$ and $g^{2n+1} = g(x)$

$f^{2012}(2009) = g^{1006}(2009) = 2009$

$f^{2009}(2010) = f[g^{1004}(2010)] = f(2010) = \frac{2005}{2007}$

$f^{2009}(2011) = f[g^{1004}(2011)] = f(2011) = \frac{2006}{2008}$

$f^{2012}(2012) = g^{1006}(2012) = 2012$

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37. (c) : Let $g(x) = \sin[f(x) - 3x]$

$g(-x) = \sin[f(-x) + 3x] = \sin[f(x) + 3x]$

38. (c) : $h(-x) = [f(-x) + f(x)][g(-x) - g(x)] = -h(x)$

39. (a) : Number of many-one functions $= 6^7$.

40. (a) : 7 can be associated to $y \in Y$ in 6 ways and 1, 2, 3, 4, 5, 6 can be associated in 5 ways each.

$\therefore$ The total number of functions required $= 6(5)^6$.

41. A → (t); B → (t); C → (q); D → (q)

(A) Given $f(0) = 2$, $f(1) = 3$

and $f(x + 2) = 2f(x) - f(x + 1)$

Put $x = 0$; $f(2) = 2f(0) - f(1) = 2 \times 2 - 3 = 1$

Put $x = 1$; $f(3) = 2f(1) - f(2) = 2 \times 3 - 1 = 5$

Put $x = 2$; $f(4) = 2f(2) - f(3) = 2 \times 1 - 5 = -3$

Put $x = 3$; $f(5) = 2f(3) - f(4) = 2 \times 5 - (-3) = 13$

(B) $\sqrt{13} > 0 \Rightarrow f\left(\sqrt{13}\right) = \left(\sqrt{13}\right)^2 = 13$

(C) Given $f(x) + 2f(1-x) = x^2 + 2, \forall x \in R$ ...(i)

Replace x by $1 - x$, we get

$f(1-x) + 2f(x) = (1-x)^2 + 2$ ...(ii)

Solving (i) & (ii), we get $f(x) = \dfrac{(x-2)^2}{3}$

$\therefore$ $f(5) = 3$.

(D) Given, $f(x) = \dfrac{4^x}{4^x + 2}$

$$\Rightarrow f(1-x) = \frac{4^{1-x}}{4^{1-x}+2} = \frac{\frac{4}{4^x}}{\frac{4}{4^x}+2} = \frac{4}{4+2\cdot 4^x} = \frac{2}{4^x+2}$$

$\therefore$ $f(x) + f(1 - x) = 1$

$$\Rightarrow f\left(\frac{1}{7}\right) + f\left(\frac{6}{7}\right) = 1 ; \; f\left(\frac{2}{7}\right) + f\left(\frac{5}{7}\right) = 1$$

and $f\left(\frac{3}{7}\right) + f\left(\frac{4}{7}\right) = 1 \Rightarrow \sum_{k=1}^{6} f\left(\frac{k}{7}\right) = 1+1+1 = 3$

42. A → (p, r, s) ; B → (r,s) ; C → (t) ; D → (q, t)

(A) Put $x = y = 0$, then $2f(0) = 2\{f(0)\}^2 \Rightarrow f(0) = 1$

Now put $x = 0$, then

$f(y) + f(-y) = 2f(0)\, f(y)$

$\therefore$ $f(y) = f(-y) \Rightarrow f(x)$ is even function

$\Rightarrow$ $f(x)$ is many one and cannot be onto.

(B) If $x \le 0$ then $f(x) = 0 \Rightarrow f(x)$ is many one and neither even nor odd. Clearly, $f(x)$ is not onto.

(C) $f(x)$ is either $-x^n + 1$ or $x^n + 1$. But $f(3) = 28$

$\therefore$ $f(x) = x^3 + 1$ which is neither even nor odd but one-one and onto both.

(D) Obviously, $f(x)$ is odd and $f'(x) = 2 + \cos x > 0$, so $f(x)$ is one-one.

Also, $f(x) \to \infty$ if $x \to \infty$ and $f(x) \to -\infty$ as $x \to -\infty$ so $f(x)$ is onto.

43. (3) : From the given equation, we have

$$\left(\frac{n}{2} - \left[\frac{n}{2}\right]\right) + \left(\frac{n}{3} - \left[\frac{n}{3}\right]\right) + \left(\frac{n}{5} - \left[\frac{n}{5}\right]\right) = 0$$

$$\Rightarrow \left\{\frac{n}{2}\right\} + \left\{\frac{n}{3}\right\} + \left\{\frac{n}{5}\right\} = 0; \text{ where } \{.\} \text{ is a fractional part.}$$

But each of the fractional part function is positive and their sum is zero. Hence, each of the fractional part function is zero. Consequently, each of $\frac{n}{2}, \frac{n}{3}, \frac{n}{5}$ is an integer. The L.C.M. of 2, 3, 5 is 30. Therefore we can take $n = 30k$, where k is an integer.

Hence, the number of solutions such that $1 \le n \le 100$ is 3 (n = 30, 60 and 90).

44. (0) : From the given equation, we get

$x^2 + (x - f) - 4x + 3 = 0$, where f = fractional part function such that $0 \le f < 1$

$\Rightarrow$ $(x^2 - 3x + 3) - f = 0 \Rightarrow f = x^2 - 3x + 3$

But $0 \le f < 1$. Therefore $0 \le x^2 - 3x + 3 < 1$ ...(i)

Now, $x^2 - 3x + 3 = 0$ gives $x = \dfrac{3 \pm \sqrt{9-12}}{2}$ (imaginary)

Again from $x^2 - 3x + 3 < 1$, we get $x^2 - 3x + 2 < 0$

$\Rightarrow$ $(x - 2)(x - 1) < 0 \Rightarrow 1 < x < 2$.

Thus the inequality (i) is satisfied if $1 < x < 2$

$\Rightarrow$ $[x] = 1$.

Putting $[x] = 1$ in the given equation, we get

$x^2 + 1 - 4x + 3 = 0 \Rightarrow (x - 2)^2 = 0 \Rightarrow x = 2$

But $x = 2$ does not satisfy the inequality $1 < x < 2$.

$\therefore$ The given equation has no solution.

45. (4) : $f(g(x)) = \begin{cases} x^2 + 1, & -1 \le x \le 1 \\ 2x^2 + 1, & 1 < x \le \sqrt{2} \end{cases}$

Now, $f(g(x)) = 2 \Rightarrow x^2 + 1 = 2$ and $2x^2 + 1 = 2$

$\Rightarrow$ $x = \pm 1$ and $x = \pm \dfrac{1}{\sqrt{2}}$

$\therefore$ Number of solutions = 4.

◈◈

Monthly Test Drive-6 CLASS XII ANSWER KEY

1.	(b)	**2.**	(b)	**3.**	(b)	**4.**	(c)	**5.**	(c)
6.	(d)	**7.**	(b,c)	**8.**	(a,d)	**9.**	(a,b,c,d)	**10.**	(a,c)
11.	(d)	**12.**	(a,c)	**13.**	(b,d)	**14.**	(a)	**15.**	(b)
16.	(a)	**17.**	(124)	**18.**	(7)	**19.**	(0)	**20.**	(4)

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CBSE warm-up!

CLASS-XII

Chapterwise Practice questions for CBSE Exams as per the reduced syllabus and marking scheme issued by CBSE for the academic session 2020-21.

Series 6

Differential Equations

Time Allowed : 3 hours
Maximum Marks : 80

GENERAL INSTRUCTIONS

(1) This question paper contains two parts A and B. Each part is compulsory. Part A carries 24 marks and Part B carries 56 marks.
(2) Part-A has Objective Type Questions and Part -B has Descriptive Type Questions.
(3) Both Part A and Part B have internal choices.

Part-A :

(1) It consists of two sections- I and II.
(2) Section I comprises of 16 very short answer type questions.
(3) Section II contains 2 case studies. Each case study comprises of 5 case-based MCQs. An examinee is to attempt any 4 out of 5 MCQs.

Part-B :

(1) It consists of three sections- III, IV and V.
(2) Section III comprises of 10 questions of 2 marks each.
(3) Section IV comprises of 7 questions of 3 marks each.
(4) Section V comprises of 3 questions of 5 marks each.
(5) Internal choice is provided in 3 questions of Section-III, 2 questions of Section-IV and 3 questions of Section-V. You have to attempt only one of the alternatives in all such questions.

PART A

SECTION - I

All questions are compulsory. In case of internal choices attempt any one.

1. Write the degree of the differential equation $\left(\frac{dy}{dx}\right)^4 + 3x\frac{d^2y}{dx^2} = 0$.

OR

Write the sum of the order and degree of the differential equation $1+\left(\frac{dy}{dx}\right)^4 = 7\left(\frac{d^2y}{dx^2}\right)^3$.

2. Write the solution of the differential equation $\frac{dy}{dx} = 2^{-y}$.

3. Find the integrating factor of the differential equation $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$.

OR

Find the integrating factor of the differential equation $\frac{dy}{dx} + y\tan x - \sec x = 0$.

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4. Find the solution of the differential equation $\frac{dy}{dx} = x^3 e^{-2y}$.

5. Verify that $x + y = \tan^{-1} y$ is the solution of the differential equation $y^2 y' + y^2 + 1 = 0$.

OR

Show that $y = \sqrt{1+x^2}$ is the solution of the differential equation $y' = \frac{xy}{1+x^2}$.

6. Write the integrating factor of the differential equation $\frac{dy}{dx} + y = \frac{1+y}{x}$.

7. Find the general solution of the differential equation $\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$.

OR

Find the general solution of the differential equation $\frac{ydx - xdy}{y} = 0$ is

8. Find the degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$.

9. Write the integrating factor of $\frac{xdy}{dx} - y = x^4 - 3x$.

OR

Write the integrating factor of the differential equation

$(1-x^2)\frac{dy}{dx} - xy = 1$ is

10. Find the number of arbitrary constants in the particular solution of a differential equation of third order.

11. Find the order of the differential equation whose general solution is given by

$y = (C_1 + C_2)\cos(x + C_3) - C_4 e^{x+C_5}$, where C_1, C_2, C_3, C_4, C_5 are arbitrary constants.

12. Find the solution for the differential equation $\frac{dy}{y} + \frac{dx}{x} = 0$.

13. Find the number of arbitrary constants in the general solution of a differential equation of fourth order.

14. Find the solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

15. Find the general solution of the differential equation $(x^2+1)\frac{dy}{dx} = 1$.

16. Find the solution of $e^{\frac{dy}{dx}} = x$ when $x = 1$ and $y = 0$.

SECTION - II

Both the case study based questions are compulsory. Attempt any 4 sub-parts from each question 17 and 18. Each question carries 1 mark.

17. In a murder investigation, a corpse was found by a detective at exactly 8 p.m. Being alert, the detective also measured the body temperature and found it to be 70°F. Two hours later, the detective measured the body temperature again and found it to be 60°F, where the room temperature is 50°F and the body temperature of the person before death was 98.6°F.

Let T be the temperature of the body at any time t and initial time taken to be 8 p.m.

Based on the above information, answer the following questions.

(i) By Newton's law of cooling, $\frac{dT}{dt}$ is proportional to

(a) $T - 60$ (b) $T - 50$
(c) $T - 70$ (d) $T - 98.6$

(ii) When $t = 0$, then body temperature is equal to
(a) 50°F (b) 60°F (c) 70°F (d) 98.6°F

(iii) When $t = 2$, then body temperature is equal to
(a) 50°F (b) 60°F (c) 70°F (d) 98.6°F

(iv) The value of T at any time t is

(a) $50 + 20\left(\frac{1}{2}\right)^t$ (b) $50 + 20\left(\frac{1}{2}\right)^{t-1}$

(c) $50 + 20\left(\frac{1}{2}\right)^{t/2}$ (d) None of these

(v) If it is given that $\log_e(2.43) = 0.88789$ and $\log_e(0.5) = -0.69315$ then the time at which the murder occur is
(a) 7:30 p.m. (b) 5:30 p.m.
(c) 6:00 p.m. (d) 5:00 p.m.

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18. A rumour on whatsapp spreads in a population of 5000 people at a rate proportional to the product of the number of people who have heard it and the number of people who have not and 100 people initiate the rumour and a total of 500 people know the rumour after 2 days.

Based on the above information, answer the following questions.

(i) If $y(t)$ denote the number of people who know the rumour at any time t, then maximum value of $y(t)$ is

(a) 500 (b) 100
(c) 5000 (d) none of these

(ii) $\frac{dy}{dt}$ is proportional to

(a) $y(y - 5000)$ (b) $y(y - 500)$
(c) $y(500 - y)$ (d) $y(5000 - y)$

(iii) The value of $y(0)$ is

(a) 100 (b) 500
(c) 600 (d) 200

(iv) The value of $y(2)$ is

(a) 100 (b) 500
(c) 600 (d) 200

(v) The value of y at any time t is given by

(a) $y=\frac{5000}{e^{-5000kt}+1}$ (b) $y=\frac{5000}{1+e^{5000kt}}$

(c) $y=\frac{5000}{49e^{-5000kt}+1}$ (d) $y=\frac{5000}{49(1+e^{-5000kt})}$

PART B

SECTION - III

All questions are compulsory. In case of internal choices attempt any one.

19. Find the general solution of the equation $x\,dy + (y - x^2)\,dx = 0$.

20. If $\sqrt{y}=\cos^{-1}x$, then it satisfies the differential equation $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}=c$, . Find the value of c.

OR

If $\frac{dy}{dx}=e^{-2y}$ and $y = 0$ when $x = e$, then find the value of x when $y=\frac{1}{2}$.

21. If $2f(x) = f'(x)$ and $f(0) = 3$, then find $f(2)$.

22. If $\frac{dy}{dx}=y+3>0$ and $y(0)=2$, then find the value of $y(\ln 2)$.

23. Write the general solution of the differential equation $\log_e\left(\frac{dy}{dx}\right)=x+y$.

OR

Find the equation of the curve passing through (1, 1) and satisfying the differential equation $\frac{dy}{dx}=\frac{2y}{x}, x>0, y>0$.

24. Solve the following differential equation: $\frac{dy}{dx}+y=\cos x-\sin x$.

25. Write the degree of the differential equation $x=1+\left(\frac{dy}{dx}\right)+\frac{1}{2!}\left(\frac{dy}{dx}\right)^2+\frac{1}{3!}\left(\frac{dy}{dx}\right)^3+.....$

26. Find the general solution of the differential equation $\frac{dy}{dx}=e^{y+x}+e^{y-x}$.

27. Find the general solution of the differential equation $2x\frac{dy}{dx}-y=3$.

28. The slope of a curve at any point is the reciprocal of twice the ordinate at the point and it passes through the points (4, 3). Find the equation of the curve.

OR

The slope at any point of a curve $y = f(x)$ is given by $\frac{dy}{dx}=3x^2$ and it passes through (–1, 1). Find the equation of the curve.

SECTION - IV

All questions are compulsory. In case of internal choices attempt any one.

29. Find the particular solution of the differential equation $\frac{dy}{dx}=\frac{xy}{x^2+y^2}$, given that $y = 1$ when $x = 0$.

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30. Solve the following differential equation :

$$\left(\sqrt{1+x^2+y^2+x^2y^2}\right)dx+xy\,dy=0$$

31. Solve the following differential equation

$(\tan^{-1}x - y)dx = (1 + x^2)dy$

OR

Solve the following differential equation :

$xdy - ydx = \sqrt{x^2+y^2}\,dx$, given that $y = 0$ when $x = 1$

32. Solve the following differential equation :

$$x\cos\left(\frac{y}{x}\right)\frac{dy}{dx}=y\cos\left(\frac{y}{x}\right)+x;\ x\neq 0.$$

33. Solve the differential equation

$(x^2 - yx^2)dy + (y^2 + x^2y^2)dx = 0$, given that $y = 1$ when $x = 1$.

34. Find the particular solution of the following differential equation :

$$xy\frac{dy}{dx}=(x+2)(y+2);\ y=-1 \text{ when } x=1$$

OR

Find the particular solution of the following differential equation :

$$(x+1)\frac{dy}{dx}=2e^{-y}-1;\ y=0 \text{ when } x=0$$

35. Solve the following differential equation :

$(1 + y^2)(1 + \log x)\,dx + xdy = 0$

SECTION - V

All questions are compulsory. In case of internal choices attempt any one.

36. Show that the differential equation $\frac{dy}{dx}=\frac{y^2}{xy-x^2}$ is homogeneous and also solve it.

OR

Find the particular solution of the differential equation

$$(x-y)\frac{dy}{dx}=(x+2y),$$ given that $y = 0$ when $x = 1$.

37. Solve the following differential equation

$$\left(x\sin^2\left(\frac{y}{x}\right)-y\right)dx+xdy=0 \text{ given } y=\frac{\pi}{4} \text{ when } x=1.$$

OR

Solve the following differential equation :

$$\left[y-x\cos\left(\frac{y}{x}\right)\right]dy+\left[y\cos\left(\frac{y}{x}\right)-2x\sin\left(\frac{y}{x}\right)\right]dx=0$$

38. Find the particular solution of the differential equation $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$, given that $y = 1$ when $x = 0$.

OR

Find a particular solution of the differential equation $(x - y)(dx + dy) = dx - dy$, given that $y = -1$, when $x = 0$. (Hint: put $x - y = t$)

SOLUTIONS

1. Degree of the given differential equation is 1.

OR

Since, Order = 2, Degree = 3

$\therefore$ Order + Degree = 2 + 3 = 5

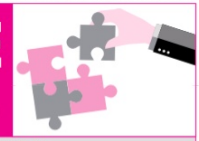
2. We have, $\frac{dy}{dx}=2^{-y}$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2^y} \quad \Rightarrow \quad 2^y\,dy=dx \qquad ...(i)$$

Integrating both sides of (i), we get

$$\frac{2^y}{\log 2}=x+C \quad \Rightarrow \quad 2^y=(C+x)\log 2$$

PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.

13+	4÷	10×		6	2÷
		5×		8+	
	30×	4	8+		
		1		12×	
4−	15+		3−		2
	2		60×		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

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Taking log on both sides to the base 2, we get
$\log_2 2^y = \log_2 [(C + x) \log 2]$
$\Rightarrow \quad y = \log_2 [(C + x) \log 2]$, which is the required solution.

3. We have, $\left(\frac{e^{-2\sqrt{x}}}{\sqrt{x}} - \frac{y}{\sqrt{x}}\right)\frac{dx}{dy} = 1$

$\Rightarrow \quad \frac{dy}{dx} + \frac{1}{\sqrt{x}}y = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$,

which is linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = \frac{1}{\sqrt{x}}$, and $Q = \frac{e^{-2\sqrt{x}}}{\sqrt{x}}$.

$\therefore \quad$ I.F. $= e^{\int P dx} = e^{\int \frac{1}{\sqrt{x}} dx} = e^{2\sqrt{x}}$

OR

We have, $\frac{dy}{dx} + y \tan x - \sec x = 0 \Rightarrow \frac{dy}{dx} + y \tan x = \sec x$

which is a linear differential equation where $P = \tan x$ and $Q = \sec x$.

$\therefore \quad$ I.F. $= e^{\int \tan x \, dx} = e^{\log \sec x} = \sec x$

4. We have, $\frac{dy}{dx} = x^3 e^{-2y} \Rightarrow e^{2y} \, dy = x^3 dx$

On integrating, we get

$\frac{e^{2y}}{2} = \frac{x^4}{4} + C'$

$\Rightarrow \quad 2 e^{2y} = x^4 + C$, where $C = 4C'$

5. We have, $x + y = \tan^{-1} y$ ...(i)

Differentiating (i) w.r.t. x, we get

$1 + y' = \frac{1}{1 + y^2}(y') \Rightarrow (1 + y')(1 + y^2) = y'$

$\Rightarrow \quad 1 + y^2 + y' + y^2 y' = y' \Rightarrow 1 + y^2 + y^2 y' = 0$

$\therefore \quad x + y = \tan^{-1} y$ is a solution of $1 + y^2 + y^2 y' = 0$

OR

We have, $y = \sqrt{1 + x^2}$ ...(i)

Differentiating (i) w.r.t. x, we get

$y' = \frac{1 \times (2x)}{2\sqrt{1 + x^2}} \Rightarrow y' = \frac{x}{\sqrt{1 + x^2}}$ ...(ii)

Dividing (ii) by (i), we get $\frac{y'}{y} = \frac{x}{1 + x^2} \Rightarrow y' = \frac{xy}{1 + x^2}$

$\therefore \quad y = \sqrt{1 + x^2}$ is a solution of $y' = \frac{xy}{1 + x^2}$.

6. We have $\frac{dy}{dx} + y = \frac{1 + y}{x} \Rightarrow \frac{dy}{dx} + y = \frac{1}{x} + \frac{y}{x}$

$\Rightarrow \quad \frac{dy}{dx} + y - \frac{y}{x} = \frac{1}{x} \Rightarrow \frac{dy}{dx} + \left(1 - \frac{1}{x}\right)y = \frac{1}{x}$

$\therefore \quad$ I.F. $= e^{\int \left(1 - \frac{1}{x}\right) dx} = e^{x - \log x} = e^x e^{-\log x} = e^x e^{\log x^{-1}}$

$= e^x x^{-1} = \frac{e^x}{x}$

7. We have, $\frac{dy}{dx} + \frac{\sqrt{1 - y^2}}{\sqrt{1 - x^2}} = 0 \Rightarrow \frac{dy}{\sqrt{1 - y^2}} + \frac{dx}{\sqrt{1 - x^2}} = 0$

Integrating both sides, we get $\int \frac{dy}{\sqrt{1 - y^2}} + \int \frac{dx}{\sqrt{1 - x^2}} = C$

$\Rightarrow \quad \sin^{-1} y + \sin^{-1} x = C$, which is the required solution.

OR

The given equation is $\frac{ydx - xdy}{y} = 0$

$\Rightarrow \quad dx - \frac{x}{y}dy = 0 \quad \Rightarrow \frac{dx}{x} - \frac{dy}{y} = 0$ ...(i)

Integrating both sides of (i), we get

$\int \frac{dx}{x} - \int \frac{dy}{y} = 0 \quad \Rightarrow \log |x| - \log |y| = \log |C'|$

$\Rightarrow \quad \log \left|\frac{x}{y}\right| = \log |C'| \quad \Rightarrow \quad \frac{x}{y} = C'$

$\Rightarrow \quad y = \frac{1}{C'}x \quad \Rightarrow y = Cx \quad \left[\text{where } \frac{1}{C'} = C\right]$,

which is the required solution.

8. Since, the given differential equation is not a polynomial in $\frac{dy}{dx}$.

Therefore, its degree is not defined.

9. We have, $\frac{xdy}{dx} - y = x^4 - 3x$

$\Rightarrow \quad \frac{dy}{dx} - \frac{y}{x} = \frac{x^4 - 3x}{x} \quad \Rightarrow \quad \frac{dy}{dx} - \frac{1}{x} \cdot y = x^3 - 3$

$\therefore \quad$ I.F. $= e^{-\int \frac{1}{x} dx} = e^{-\log x} = e^{\log x^{-1}} = x^{-1} = \frac{1}{x}$

OR

We have, $(1 - x^2)\frac{dy}{dx} - xy = 1 \Rightarrow \frac{dy}{dx} - \frac{x}{1 - x^2} \cdot y = \frac{1}{1 - x^2}$

$\therefore \quad$ I.F. $= e^{-\int \frac{x}{1 - x^2} dx} = e^{\frac{1}{2}\int \frac{-2x dx}{1 - x^2}}$

$= e^{\frac{1}{2}\log(1 - x^2)} = e^{\log(1 - x^2)^{\frac{1}{2}}} = \sqrt{1 - x^2}$

10. In the particular solution of a differential equation of any order there is no arbitrary constant.

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11. The given equation can be written as
$y = A\cos(x + C_3) - Be^x$,
where $A = C_1 + C_2$ and $B = C_4 e^{C_5}$
So, there are three independent variables, (A, B, C_3).
Hence, the differential equation will be of order 3.

12. We have, $\frac{dy}{y} + \frac{dx}{x} = 0 \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$
Integrating both sides, we get
$\Rightarrow \int \frac{dy}{y} = -\int \frac{dx}{x} \Rightarrow \log y = -\log x + \log c$
$\Rightarrow \log(yx) = \log c \Rightarrow xy = c$

13. The number of arbitrary constants in the general solution of a differential equation of n^{th} order is n. So, differential equation of fourth order has 4 constants.

14. From the given differential equation, we have
$\frac{dy}{dx} = \frac{e^x + x^2}{e^y} \Rightarrow e^y dy = (e^x + x^2)\, dx$
Integrating both sides, we get
$e^y = e^x + \frac{x^3}{3} + c$

15. We have, $(x^2 + 1)\frac{dy}{dx} = 1 \Rightarrow dy = \frac{dx}{x^2 + 1}$
Integrating both sides, we get
$\int dy = \int \frac{dx}{x^2 + 1} \Rightarrow y = \tan^{-1}(x) + C$

16. We have, $e^{\frac{dy}{dx}} = x \Rightarrow \frac{dy}{dx} = \log x \Rightarrow \int dy = \int (\log x) dx$
$\Rightarrow y = x \cdot \log x - x + C$
Putting $x = 1$ and $y = 0$, we get, $C = 1$
$\therefore \quad y = x \log x - x + 1$

17. **(i) (b) :** Given T be the temperature of the body at any time t and initial time O taken to be 8 p.m. Then, by Newton's law of cooling, we get
$\frac{dT}{dt} = k(T - 50)$, where k is the constant of proportionality.
(ii) (c) : From given information, we have
At 8 p.m. temperature is 70°F
$\therefore$ At $t = 0$, $T = 70°$F
(iii) (b) : From given information, we have
At 10 p.m., temperature is 60°F
$\therefore$ At $t = 2$, $T = 60°$ F
(iv) (c) : $\frac{dT}{dt} = k(T - 50) \Rightarrow \frac{dT}{T - 50} = k\, dt$
On integrating both sides, we get
$\log|T - 50| = kt + \log C$
$\Rightarrow \quad T - 50 = Ce^{kt}$
Clearly, for $t = 0$, $T = 70° \Rightarrow C = 20$
Thus, $T - 50 = 20e^{kt}$
For $t = 2$, $T = 60° \Rightarrow 10 = 20\, e^{2k}$
$\Rightarrow \quad 2k = \log\left(\frac{1}{2}\right) \Rightarrow k = \frac{1}{2}\log\left(\frac{1}{2}\right)$
Hence, $T = 50 + 20\left(\frac{1}{2}\right)^{\frac{t}{2}}$
(v) (d) : We have, $T = 50 + 20\left(\frac{1}{2}\right)^{t/2}$
Now, $98.6 = 50 + 20\left(\frac{1}{2}\right)^{\frac{t}{2}}$
$\Rightarrow \quad \frac{48.6}{20} = \left(\frac{1}{2}\right)^{\frac{t}{2}} \Rightarrow \log\left(\frac{48.6}{20}\right) = \frac{t}{2}\log\left(\frac{1}{2}\right)$
$\Rightarrow \quad \frac{t}{2} = \frac{\log\left(\frac{48.6}{20}\right)}{\log\left(\frac{1}{2}\right)} \quad \Rightarrow \quad t = 2\left(\frac{\log\left(\frac{48.6}{20}\right)}{\log\left(\frac{1}{2}\right)}\right) \approx -2.56$
So, it appears that the person was murdered 2.5 hours before 8 p.m. *i.e.*, about 5 : 30 p.m.

18. **(i) (c) :** Since, size of population is 5000.
$\therefore$ Maximum value of $y(t)$ is 5000.
(ii) (d) : Clearly, according to given information, $\frac{dy}{dt} = ky(5000 - y)$, where k is the constant of proportionality.
(iii) (a) : Since, rumour is initiated with 100 people.
$\therefore$ When $t = 0$, then $y = 100$
Thus $y(0) = 100$
(iv) (b) : Since, rumour is spread in 500 people, after 2 days.
$\therefore$ When $t = 2$, then $y = 500$.
Thus, $y(2) = 500$
(v) (c) : We know that, when $t = 0$, then $y = 100$
This condition is satisfied by option (c) only.

19. The given equation is $xdy + (y - x^2)\, dx = 0$
$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = x$, ...(i)
which is a linear differential equation of the type $\frac{dy}{dx} + Py = Q$
Here $P = \frac{1}{x}$ and $Q = x \quad \therefore \quad \text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\log|x|} = x$
$\therefore$ The solution is $y(\text{I.F.}) = \int Q(\text{I.F.})\, dx + C$
$\Rightarrow \quad yx = \int x.x\, dx + C \Rightarrow yx = \int x^2\, dx + C$

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$\Rightarrow \quad yx = \frac{1}{3}x^3 + C \Rightarrow y = \frac{x^2}{3} + \frac{C}{x}$,

which is the required solution.

20. We have, $\sqrt{y} = \cos^{-1} x$

$\therefore \quad y = (\cos^{-1} x)^2 \quad \Rightarrow \quad y_1 = 2(\cos^{-1} x)\left\{-\frac{1}{\sqrt{1-x^2}}\right\}$

$\Rightarrow \quad (1-x^2)y_1^2 = 4(\cos^{-1} x)^2 = 4y$

$\Rightarrow \quad (1-x^2)\cdot 2y_1y_2 - 2xy_1^2 = 4y_1$

$\left[\text{Here, } y_1 = \frac{dy}{dx} \text{ and } y_2 = \frac{d^2y}{dx^2}\right]$

$\Rightarrow \quad (1-x^2)y_2 - xy_1 = 2 \Rightarrow c = 2$

OR

Consider, $\frac{dy}{dx} = e^{-2y}$

Now, $\frac{dy}{e^{-2y}} = dx \Rightarrow e^{2y}\,dy = dx$

Integrating both sides, we get $\int e^{2y}\,dy = \int dx$

$\frac{e^{2y}}{2} = x + c \Rightarrow e^{2y} = 2x + 2c$ ...(i)

$\because \quad y(e) = 0$ [Given] $\quad \therefore \quad e^0 = 2e + 2c \Rightarrow c = \frac{1-2e}{2}$

Substituting the value of c in (i), we get

$e^{2y} = 2x + 1 - 2e$

Now, $y(x) = \frac{1}{2}$

$\therefore \quad e^{2\cdot\frac{1}{2}} = 2x + 1 - 2e \Rightarrow 3e - 1 = 2x \Rightarrow x = \frac{3e-1}{2}$

21. Given, $2f(x) = f'(x) \Rightarrow \frac{f'(x)}{f(x)} = 2$

Integrating both sides, we get

$\int \frac{f'(x)}{f(x)}dx = \int 2dx \Rightarrow \log_e f(x) = 2x + c$ ... (i)

Now, we have, $f(0) = 3 \Rightarrow \log_e f(0) = 2(0) + c \Rightarrow c = \log_e 3$

$\therefore \quad f(x) = e^{2x+c} = e^{2x+\log_e 3}$ [From (i)]

$\therefore \quad f(2) = e^4 . e^{\log_e 3} = 3e^4$

22. Consider, $\frac{dy}{dx} = y + 3 \quad \Rightarrow \quad \frac{dy}{y+3} = dx$

On integrating, we get

$\ln(y + 3) = x + C$

Since $y(0) = 2$, [Given]

$C = \ln 5$

Now, $\ln(y + 3) = x + \ln 5$

When $x = \ln 2$, then we have

$\ln(y + 3) = \ln 2 + \ln 5 = \ln 10$

$\Rightarrow \quad y + 3 = 10 \quad \Rightarrow \quad y = 7$

23. We have, $\log_e\left(\frac{dy}{dx}\right) = x + y$

$\therefore \quad \frac{dy}{dx} = e^{x+y} = e^x \cdot e^y \Rightarrow \quad e^x dx = e^{-y} \cdot dy$

On integrating, $e^x = -e^{-y} + C \Rightarrow \quad e^x + e^{-y} = C$

OR

We have, $\frac{dy}{dx} = \frac{2y}{x} \Rightarrow \quad \frac{dy}{y} = \frac{2}{x}dx$

On integrating, we get

$\int \frac{dy}{y} = \int \frac{2}{x}dx$

$\Rightarrow \quad \log y = 2\log x + \log C \Rightarrow \log y = \log x^2 + \log C$

$\Rightarrow \quad y = Cx^2$ ...(i)

Since, the curve passes through (1, 1) *i.e.*, $x = 1$ and $y = 1$

$\therefore \quad C = 1$

Putting value of C in (i), we get

$y = x^2$, which is the required equation of curve.

24. We have, $\frac{dy}{dx} + y = \cos x - \sin x$, which is a linear differential equation of the form $\frac{dy}{dx} + Py = Q$, where $P = 1$, $Q = \cos x - \sin x$.

$\therefore \quad$ I.F. $= e^{\int dx} = e^x$

The required solution of given differential equation is

$ye^x = \int e^x(\cos x - \sin x)dx + C$

$\Rightarrow \quad ye^x = e^x\cos x + C \Rightarrow \quad y = \cos x + Ce^{-x}$

25. We have, $x = 1 + \left(\frac{dy}{dx}\right) + \frac{1}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{1}{3!}\left(\frac{dy}{dx}\right)^3 + ...$

$\Rightarrow \quad x = e^{\frac{dy}{dx}} \quad \Rightarrow \quad \frac{dy}{dx} = \log_e x$

$\Rightarrow$ Degree of differential equation is 1.

26. We have, $\frac{dy}{dx} = e^{y+x} + e^{y-x} = e^y(e^x + e^{-x})$

$\Rightarrow \quad e^{-y}dy = (e^x + e^{-x})\,dx$

Integrating both sides, we get

$\int e^{-y}dy = \int (e^x + e^{-x})\,dx \Rightarrow \quad -e^{-y} = e^x - e^{-x} + c_1$

$\Rightarrow \quad e^{-y} = e^{-x} - e^x + c$, where $-c_1 = c$

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27. We have, $2x\frac{dy}{dx} - y = 3$

$\Rightarrow \quad 2x\frac{dy}{dx} = 3 + y \Rightarrow \frac{dy}{3+y} = \frac{dx}{2x}$

Integrating both sides, we get

$\int \frac{dy}{3+y} = \int \frac{dx}{2x}$

$\Rightarrow \quad \log(3 + y) = \log(2x) + \log c \quad \Rightarrow \quad 3 + y = 2xc$

$\Rightarrow \quad y = 2xc - 3$, which is the required general solution.

28. Given, $\frac{dy}{dx} = \frac{1}{2y}$

$\Rightarrow \int 2y\,dy = \int dx \Rightarrow \quad y^2 = x + c$ ...(i)

It is given that curve passes through (4, 3).

$\Rightarrow \quad 3^2 = 4 + c \Rightarrow c = 5$

Hence, equation of curve is $y^2 = x + 5$.

OR

We have, $\frac{dy}{dx} = 3x^2 \quad \Rightarrow \quad dy = 3x^2 dx$

Integrating both sides, we get

$\int dy = \int 3x^2 dx \Rightarrow y = x^3 + c$

$\because$ The curve passes through $(-1, 1)$

$\therefore \quad 1 = -1 + c \Rightarrow \quad c = 2$

$\therefore$ Equation of the curve is $y = x^3 + 2$.

29. We have, $\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$

This is a homogeneous linear differential equation.

Put $y = vx \Rightarrow \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore \quad v + x\frac{dv}{dx} = \frac{x \cdot vx}{x^2 + v^2x^2}$

$\Rightarrow \quad v + x\frac{dv}{dx} = \frac{v}{1+v^2} \quad \Rightarrow \quad x\frac{dv}{dx} = \frac{v}{1+v^2} - v$

$\Rightarrow \quad x\frac{dv}{dx} = \frac{-v^3}{1+v^2} \quad \Rightarrow \quad \frac{dx}{x} = -\left(\frac{1+v^2}{v^3}\right)dv$

Integrating both sides, we get

$\int \frac{dx}{x} = -\int v^{-3}dv - \int \frac{1}{v}dv$

$\Rightarrow \log x = \frac{1}{2v^2} - \log v + C$

$\Rightarrow \log x = \frac{x^2}{2y^2} - \log y + \log x + C$

$\Rightarrow \log y = \frac{x^2}{2y^2} + C$

When $y = 1, x = 0 \quad \Rightarrow \quad \log 1 = 0 + C \Rightarrow \quad C = 0$

$\therefore \quad$ Particular solution is $y = e^{\frac{x^2}{2y^2}}$.

30. We have, $\sqrt{1 + x^2 + y^2 + x^2y^2} + xy\frac{dy}{dx} = 0$

$\Rightarrow \quad \frac{dy}{dx} = \frac{-\sqrt{(1+x^2)(1+y^2)}}{xy}$

$\Rightarrow \quad \int \frac{y}{\sqrt{1+y^2}}dy = -\int \frac{\sqrt{1+x^2}}{x^2}xdx$

$\Rightarrow \quad \frac{1}{2}\int \frac{2y}{\sqrt{1+y^2}}dy = -\int \frac{v^2}{v^2-1}dv$

[Putting $1 + x^2 = v^2 \Rightarrow 2xdx = 2vdv$]

$\Rightarrow \quad \sqrt{1+y^2} = -\int\left(1 + \frac{1}{v^2-1}\right)dv$

$\Rightarrow \quad \sqrt{1+y^2} = -v - \frac{1}{2}\log\left|\frac{v-1}{v+1}\right| + C$

$\Rightarrow \quad \sqrt{1+y^2} + \sqrt{1+x^2} + \frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right| = C$

31. We have, $\frac{dy}{dx} = \frac{(\tan^{-1}x - y)}{1+x^2}$

$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{1+x^2} = \frac{\tan^{-1}x}{1+x^2}$,

which is a linear differential equation,

where $P = \frac{1}{1+x^2}$ and $Q = \frac{\tan^{-1}x}{1+x^2}$.

$\text{I.F} = e^{\int Pdx} = e^{\int \frac{1}{1+x^2}dx} = e^{\tan^{-1}x}$

$\therefore \quad$ The required solution is

$ye^{\tan^{-1}x} = \int \frac{\tan^{-1}x}{1+x^2} \cdot e^{\tan^{-1}x}dx$ ...(i)

Let $I = \int \frac{\tan^{-1}x}{1+x^2}e^{\tan^{-1}x}dx$

Put $\tan^{-1}x = t \quad \Rightarrow \quad \frac{dx}{1+x^2} = dt$

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$\therefore\ I = \int t \cdot e^t dt = t\int e^t dt - \int\left(\frac{d}{dt}(t)\int e^t dt\right)dt$

$= te^t - \int e^t dt = te^t - e^t + C = e^t(t-1) + C$

$\Rightarrow\ I = e^{\tan^{-1}x}(\tan^{-1}x - 1) + C$ ...(ii)

Putting (ii) in (i), we get

$ye^{\tan^{-1}x} = e^{\tan^{-1}x}(\tan^{-1}x - 1) + C$

$\Rightarrow\ y = \tan^{-1}x - 1 + Ce^{-\tan^{-1}x}$

OR

Refer to Answer 31, page no. 211, MTG CBSE Champion Mathematics, Class 12.

32. We have, $x\cos\left(\frac{y}{x}\right)\frac{dy}{dx} = y\cos\left(\frac{y}{x}\right) + x; x \neq 0$

$\Rightarrow \cos\left(\frac{y}{x}\right)\frac{dy}{dx} = \left(\frac{y}{x}\right)\cos\left(\frac{y}{x}\right) + 1$ ...(i)

This is a linear homogeneous differential equation.

Put $y = vx\ \Rightarrow \frac{dy}{dx} = v\cdot 1 + x\cdot\frac{dv}{dx}$

Now (i) becomes, $\cos v.\left[v + x\frac{dv}{dx}\right] = v\cos v + 1$

$\Rightarrow\ x\cos v\frac{dv}{dx} = 1\ \Rightarrow\ \cos v\, dv = \frac{dx}{x}$

Integrating both sides, we get

$\sin v = \log x + C \Rightarrow \sin\left(\frac{y}{x}\right) = \log x + C$, which is the required solution.

33. *Refer to Answer 52, page no. 216 of MTG CBSE Champion Mathematics, Class 12.*

34. We have, $xy\frac{dy}{dx} = (x+2)(y+2)$

$\Rightarrow \frac{y\,dy}{(y+2)} = \left(\frac{x+2}{x}\right)dx \Rightarrow dy - \frac{2}{(y+2)}dy = dx + \frac{2}{x}dx$

Integrating both sides, we get

$y - 2\log(y+2) = x + 2\log x + C$

Now, when $x = 1, y = -1$, we have

$-1 - 2\log(-1+2) = 1 + 2\log 1 + C$

$\Rightarrow\ C = -1 - 1 = -2$

So, we have $y - 2\log(y+2) = x + 2\log x - 2$

$\Rightarrow\ y - x + 2 = 2\log x(y+2)$

OR

We have, $(x+1)\frac{dy}{dx} = 2e^{-y} - 1$ ...(i)

$\Rightarrow \frac{dy}{2e^{-y}-1} = \frac{dx}{x+1} \Rightarrow \frac{e^y}{2-e^y}dy = \frac{dx}{x+1}$

Integrating both sides, we get

$-\log(2-e^y) = \log(x+1) + C$ ...(ii)

When $x = 0, y = 0$, we have

$\therefore\ -\log(2-1) = \log(0+1) + C \Rightarrow C = 0$

$\therefore$ Eq. (ii) becomes

$-\log(2-e^y) = \log(x+1)$

$\Rightarrow\ \log(x+1) + \log(2-e^y) = 0$

$\Rightarrow\ \log(x+1)(2-e^y) = 0\ \Rightarrow\ (x+1)(2-e^y) = 1$,

which is the required particular solution.

35. We have, $(1+y^2)(1+\log x)dx + xdy = 0$

$\Rightarrow\ (1+y^2)(1+\log x)dx = -xdy$

$\Rightarrow \left(\frac{1+\log x}{x}\right)dx = \frac{-dy}{1+y^2}$

Integrating both sides, we get

$\frac{(1+\log x)^2}{2} = -\tan^{-1}y + C$

$\Rightarrow\ \frac{(1+\log x)^2}{2} + \tan^{-1}y = C$,

which is the required general solution.

36. We have, $\frac{dy}{dx} = \frac{y^2}{xy - x^2} = \frac{y^2/x^2}{(xy-x^2)/x^2}$, ... (i)

which is homogeneous differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v\cdot 1 + x\frac{dv}{dx}$

$\therefore$ Equation (i) becomes

$v + x\frac{dv}{dx} = \frac{v^2}{v-1} \Rightarrow x\frac{dv}{dx} = \frac{v^2}{v-1} - v = \frac{v}{v-1}$

$\Rightarrow \frac{v-1}{v}dv = \frac{dx}{x} \Rightarrow \left(1 - \frac{1}{v}\right)dv = \frac{dx}{x}$

Integrating, we get

$v - \log v = \log x + C\ \Rightarrow\ v = \log vx + C$

$\Rightarrow\ \frac{y}{x} = \log y + C$

$\Rightarrow\ y = x(\log y + C)$ is the required solution.

OR

Refer to Answer 82, page no. 222 of MTG CBSE Champion Mathematics, Class 12.

37. We have, $\left(x\sin^2\left(\frac{y}{x}\right) - y\right)dx + x\,dy = 0$

$\Rightarrow\ \frac{dy}{dx} + \sin^2\left(\frac{y}{x}\right) - \frac{y}{x} = 0$, ...(i)

which is homogeneous differential equation.

Put $y = vx \Rightarrow \frac{dy}{dx} = v\cdot 1 + x\frac{dv}{dx}$ $\therefore$ (i) becomes

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$v + x\frac{dv}{dx} + \sin^2 v - v = 0$

$\Rightarrow x\frac{dv}{dx} + \sin^2 v = 0 \Rightarrow \text{cosec}^2 v dv + \frac{dx}{x} = 0$

Integrating both sides, we get

$\int \text{cosec}^2 v dv + \int \frac{dx}{x} = C \Rightarrow -\cot v + \log x = C$

$\Rightarrow -\cot\left(\frac{y}{x}\right) + \log x = C$ ...(ii)

Put $x = 1, y = \pi/4$ in (ii), we get

$-\cot\frac{\pi}{4} + \log 1 = C \Rightarrow C = -1$

$\therefore -\cot\left(\frac{y}{x}\right) + \log x + 1 = 0$, which is required particular solution.

OR

Refer to Answer 86, page no. 223 of MTG CBSE Champion Mathematics, Class 12.

38. The given equation is $(1 + e^{2x})dy + (1 + y^2)e^x dx = 0$

$\Rightarrow \frac{dy}{1+y^2} + \frac{e^x}{1+e^{2x}}dx = 0$

Integrating both sides, we get $\int \frac{dy}{1+y^2} + \int \frac{e^x}{1+e^{2x}}dx = C_1$

$\Rightarrow \tan^{-1} y + I = C_1$, ...(i)

where $I = \int \frac{e^x}{1+e^{2x}}dx$

Put $e^x = t \Rightarrow e^x dx = dt$

$\therefore I = \int \frac{dt}{1+t^2} = \tan^{-1} t + C_2 = \tan^{-1} e^x + C_2$

From (i), $\tan^{-1} y + \tan^{-1} e^x + C_2 = C_1$

$\Rightarrow \tan^{-1} y + \tan^{-1} e^x = C$ ...(ii)

[where $C = C_1 - C_2$]

When $x = 0, y = 1$, we have $\tan^{-1}(1) + \tan^{-1}(e^0) = C$

$\Rightarrow \tan^{-1}(1) + \tan^{-1}(1) = C \Rightarrow \frac{\pi}{4} + \frac{\pi}{4} = C \Rightarrow C = \frac{\pi}{2}$

Putting $C = \frac{\pi}{2}$ in (ii), we get, $\tan^{-1} y + \tan^{-1} e^x = \pi/2$, which is the required solution.

OR

The given equation is $(x - y)(dx + dy) = dx - dy$

$\Rightarrow (x - y - 1)dx + (x - y + 1)dy = 0$

$\Rightarrow \frac{dy}{dx} = -\frac{x - y - 1}{x - y + 1}$ ...(i)

Put $x - y = t \Rightarrow 1 - \frac{dy}{dx} = \frac{dt}{dx} \Rightarrow \frac{dy}{dx} = 1 - \frac{dt}{dx}$

$\therefore$ From (i), $1 - \frac{dt}{dx} = -\frac{t-1}{t+1}$

$\Rightarrow \frac{dt}{dx} = 1 + \frac{t-1}{t+1} \Rightarrow \frac{dt}{dx} = \frac{2t}{t+1} \Rightarrow \frac{t+1}{t}dt = 2dx$

Integrating both sides, we get $\int\left(1 + \frac{1}{t}\right)dt = 2\int dx + C$

$\Rightarrow t + \log|t| = 2x + C$

$\Rightarrow x - y + \log|x - y| = 2x + C$ ...(ii)

When $x = 0, y = -1$, we have $0 + 1 + \log|0 + 1| = 2(0) + C$

$\Rightarrow C = 1$

Putting $C = 1$ in (ii), we get, $x - y + \log|x - y| = 2x + 1$

$\Rightarrow \log|x - y| = x + y + 1$, which is the required solution.

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MONTHLY TEST DRIVE

XII

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 6 : Indefinite Integration

Time Taken : 60 Min.

Only One Option Correct Type

1. If $I=\int\frac{dx}{x^3\sqrt{x^2-1}}$, then I equals to

(a) $\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x}+\tan^{-1}\sqrt{x^2-1}\right)+C$

(b) $\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x^2}+\tan^{-1}\sqrt{x^2-1}\right)+C$

(c) $\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x^3}+\tan^{-1}\sqrt{x^2-1}\right)+C$

(d) $\frac{1}{2}\left(\frac{\sqrt{x^2-1}}{x^2}+x\tan^{-1}\sqrt{x^2-1}\right)+C$

2. Let $f(x)=\int\frac{x^2dx}{(1+x^2)(1+\sqrt{1+x^2})}$ and $f(0)=0$, then $f(1)=$

(a) $\log(1+\sqrt{2})$ (b) $\log(1+\sqrt{2})-\frac{\pi}{4}$

(c) $\log(1+\sqrt{2})+\frac{\pi}{2}$ (d) None of these

3. $\int\frac{x^2(1-\ln x)}{(\ln x)^4-x^4}dx$ is equal to

(a) $\frac{1}{2}\ln\left(\frac{x}{\ln x}\right)-\frac{1}{4}\ln(\ln^2 x-x^2)+C$

(b) $\frac{1}{4}\ln\left(\frac{\ln x-x}{\ln x+x}\right)-\frac{1}{2}\tan^{-1}\left(\frac{\ln x}{x}\right)+C$

(c) $\frac{1}{4}\ln\left(\frac{\ln x+x}{\ln x-x}\right)+\frac{1}{2}\tan^{-1}\left(\frac{\ln x}{x}\right)+C$

(d) $\frac{1}{4}\ln\left(\frac{\ln x-x}{\ln x+x}\right)+\frac{1}{2}\tan^{-1}\left(\frac{\ln x}{x}\right)+C$

4. For any natural number m,
$\int(x^{7m}+x^{2m}+x^m)(2x^{6m}+7x^m+14)^{1/m}dx$,
where $x>0$ equals

(a) $\frac{(7x^{7m}+2x^{2m}+14x^m)^{(m+1)/m}}{14(m+1)}+C$

(b) $\frac{(2x^{7m}+14x^{2m}+7x^m)^{(m+1)/m}}{14(m+1)}+C$

(c) $\frac{(2x^{7m}+7x^{2m}+14x^m)^{(m+1)/m}}{14(m+1)}+C$

(d) $\frac{(7x^{7m}+2x^{2m}+x^m)^{(m+1)/m}}{14(m+1)}+C$

5. If $I=\int\frac{\tan x}{\sqrt{a+b\tan^2 x}}dx(a>b)$, then I equals

(a) $\frac{1}{\sqrt{b-a}}\sin^{-1}\left(\sqrt{\frac{a+b\tan^2 x}{b-a}}\right)+C$

(b) $\frac{1}{\sqrt{b-a}}\cos^{-1}\left(\sqrt{\frac{a+b\tan^2 x}{b-a}}\right)+C$

(c) $\frac{1}{\sqrt{b-a}}\tan^{-1}\left(\sqrt{\frac{a+b\tan^2 x}{b-a}}\right)+C$

(d) $\frac{1}{\sqrt{b-a}}\tan^{-1}\left(\sqrt{\frac{b-a}{a+b\tan^2 x}}\right)+C$

6. If $f(x)=\frac{x+2}{2x+3}$. Then, $\int\left\{\frac{f(x)}{x^2}\right\}\frac{1}{2}dx$ is equal to

$\frac{1}{\sqrt{2}}g\left\{\frac{1+\sqrt{2}f(x)}{1-\sqrt{2}f(x)}\right\}-\sqrt{\frac{2}{3}}h\left\{\frac{\sqrt{3f(x)}+\sqrt{2}}{\sqrt{3f(x)}-\sqrt{2}}\right\}+C$, where

(a) $g(x)=\tan^{-1}x$, $h(x)=\log|x|$

(b) $g(x)=\log|x|$, $h(x)=\tan^{-1}x$

(c) $g(x)=h(x)=\tan^{-1}x$

(d) $g(x)=\log|x|$, $h(x)=\log|x|$

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One or More Than One Option(s) Correct Type

7. If $I = \int \log(\sqrt{x-a} + \sqrt{x-b})dx$, then I equals

(a) $[2x-(a+b)]\log(\sqrt{x-a}+\sqrt{x-b})]+C$

(b) $\frac{1}{2}[2x-(a+b)](\log(b-a)-\log(\sqrt{x-a}-\sqrt{x-b}))$
$-\left(\frac{1}{2}\right)\sqrt{(x-a)(x-b)}+C$

(c) $\frac{1}{2}[2x-(a+b)]\log(\sqrt{x-a}+\sqrt{x-b})$
$-\frac{1}{2}\sqrt{(x-a)(x-b)}+C$

(d) None of these

8. If $\int \sin^{-1} x \cos^{-1} x\, dx = f^{-1}(x) \times \left[Ax - xf^{-1}(x) - 2\sqrt{1-x^2}\right] + A\sqrt{1-x^2} + 2x + C$, then

(a) $f(x) = \sin x$ (b) $f(x) = \cos x$

(c) $A = \pi/4$ (d) $A = \pi/2$

9. If $\int \frac{x + \sqrt[3]{x^2} + \sqrt[6]{x}}{x(1+\sqrt[3]{x})}dx = \frac{A}{2}\sqrt[3]{x^2} + B\tan^{-1}\sqrt[6]{x} + C$, then

(a) $A + B = 9$ (b) $A = 3$

(c) $B = 6$ (d) $B - A = 3$

10. If $\int xe^{-5x^2} \sin 4x^2 dx = Ke^{-5x^2}(A\sin 4x^2 + B\cos 4x^2) + C$, then

(a) $K = -1/82$ (b) $K = 1/82$

(c) $A = 5$ (d) none of these

11. If $I = \int \frac{\sin^3(\theta/2)}{\cos(\theta/2)\sqrt{\cos^3\theta + \cos^2\theta + \cos\theta}}d\theta$, then I equals

(a) $\cot^{-1}(\tan\theta + \sec\theta) + C$

(b) $\cot^{-1}(\cos\theta + \sin\theta + 1) + C$

(c) $\tan^{-1}\left(\tan\frac{\theta}{2} + \sec\frac{\theta}{2} + 1\right) + C$

(d) $\tan^{-1}(\cos\theta + \sec\theta + 1) + C$

12. If $\int \frac{(2x-1)dx}{x^4 - 2x^3 + x + 1} = A\tan^{-1}\left[\frac{f(x)}{\sqrt{3}}\right] + C$, then

(a) $A = \frac{2}{\sqrt{3}}$ (b) $A = 2$

(c) $f(x) = 2x^2 - 2x - 1$ (d) $f(x) = 2x^2 + 2x + 1$

13. If $\int \sqrt{\text{cosec}\, x + 1}\, dx = kfog(x) + c$, where k is a real constant, then

(a) $k = -2, f(x) = \cot^{-1} x, g(x) = \sqrt{\text{cosec}\, x - 1}$

(b) $k = -2, f(x) = \tan^{-1} x, g(x) = \sqrt{\text{cosec}\, x - 1}$

(c) $k = 2, f(x) = \tan^{-1} x, g(x) = \frac{\cot x}{\sqrt{\text{cosec}\, x - 1}}$

(d) $k = 2, f(x) = \cot^{-1} x, g(x) = \frac{\cot x}{\sqrt{\text{cosec}\, x + 1}}$

Comprehension Type

If A is square matrix and e^A is defined as $e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \ldots = \frac{1}{4}\begin{bmatrix} f(x) & g(x) \\ g(x) & f(x) \end{bmatrix}$, where $A = \begin{bmatrix} x & x \\ x & x \end{bmatrix}$ and $0 < x < 1$, I is an identity matrix.

14. $\int \frac{g(x)}{f(x)}dx$ is equal to

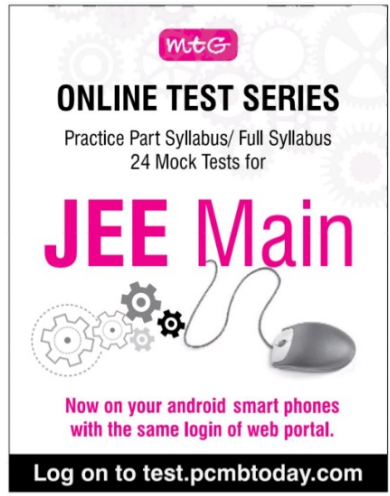
(a) $\log(e^x + e^{-x}) + c$ (b) $\log|e^x - e^{-x}| + c$

(c) $\log|e^{2x} - 1| + c$ (d) None of these

15. $\int (g(x) + 1)\sin x dx$ is equal to

(a) $\frac{e^x}{2}(\sin x - \cos x)$ (b) $\frac{e^{2x}}{5}(2\sin x - \cos x)$

(c) $\frac{e^x}{5}(\sin 2x - \cos 2x)$ (d) None of these



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Matrix Match Type

16. Match the following :

	Column-I	Column-II	
P.	If $\int \frac{2^x}{\sqrt{1-4^x}}dx = (\log k)^{-1}\sin^{-1}(f(x)) + C$, then k is greater than	1.	0
Q.	If $\int \frac{(\sqrt{x})^5}{(\sqrt{x})^7 + x^6}dx = a\ln\frac{x^k}{x^k+1} + c$, then ak is less than	2.	1
R.	If $\int \frac{x^4+1}{x(x^2+1)^2}dx = k\ln\lvert x\rvert + \frac{m}{1+x^2} + c$, then mk is greater than	3.	3
S.	If $\int \frac{dx}{5+4\cos x} = k\tan^{-1}\left(m\tan\frac{x}{2}\right) + C$, then k/m is greater than	4.	4

	P	Q	R	S
(a)	1, 2	3, 4	1	1, 2
(b)	3, 4	2	1	1, 3
(c)	1	2	3	4
(d)	1, 2	1, 3	3	4

Numerical Answer Type

17. If $\int \frac{2e^{5x}+e^{4x}-4e^{3x}+4e^{2x}+2e^x}{(e^{2x}+4)(e^{2x}-1)^2}dx = \tan^{-1}\left(\frac{e^x}{2}\right) - \frac{K}{248(e^{2x}-1)} + C$, then K is equal to

18. If $I = \int \frac{dx}{1+\sqrt{x^2+2x+2}}$

$= \frac{A}{7}\log\lvert x+1+\sqrt{x^2+2x+2}\rvert - \frac{\sqrt{x^2+2x+2}-1}{x+1} + C$

then A is equal to

19. If $f(x) = \sqrt{x}$, $g(x) = e^x - 1$, and $\int fog(x)dx = A\,fog(x) + B\tan^{-1}(fog(x)) + C$, then $A + B$ is equal to

20. Let $g(x) = \int \frac{1+2\cos x}{(\cos x+2)^2}dx$ and $g(0) = 0$, then the value of $8g(\pi/2)$ is

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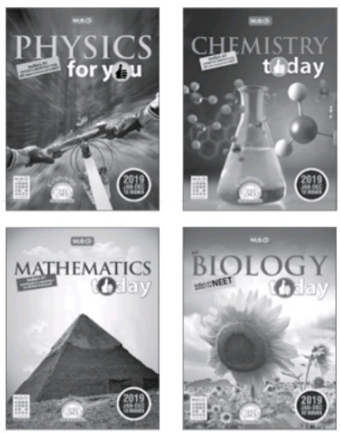

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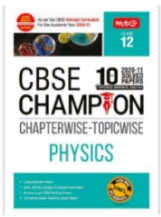

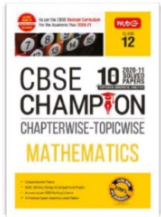

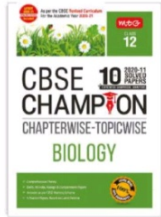

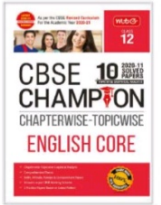

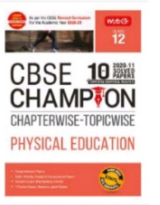

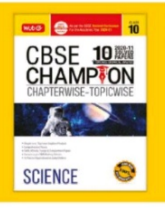

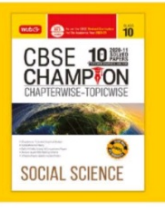

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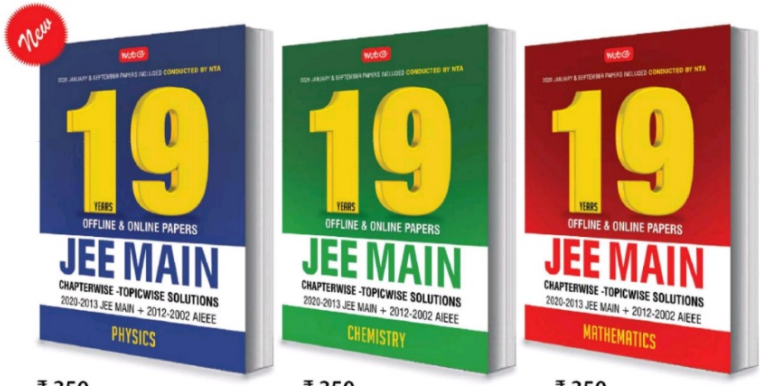

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